Application oriented vehicle problems in public bus transportation

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Joint work with

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Szeged

The city of Szeged

Szeged
The University of Szeged

Szeged

Public transportation in Szeged

Motivation

- R&D Project

Overview

- Scheduling problems in public transportation
- Vehicle scheduling
- Vehicle assignment
- „Driver-friendly” vehicle schedules
- Vehicle rescheduling

Scheduling problems in public transportation

For public transportation services certain number of stations, and previously determined – bus or other vehicles – lines are given. Each line connects a pair of stations.

The lines are fixed in timetables which provides the departure and arrival time of the trips for each line, and – sometimes – further services for each day are also fixed.

One of the most important subject of a public transport company is to minimize its operational costs.

Scheduling problems in public transportation (Operations planning)

Routing

↓

Line frequencies

↓

Timetable

↓

Scheduling

„Outside controled”

Scheduling problems in public transportation

This complex problem is divided into subtasks and they are to be solved as separated optimization problems

Vehicle scheduling

↓

Driver scheduling

↓

Driver rostering

Vehicle Scheduling Problem

Vehicle scheduling problem

Input:

- Timetabled trips
- Deadhead trips
- Depots
  - Bus types (solo, double, etc.)
  - Locations

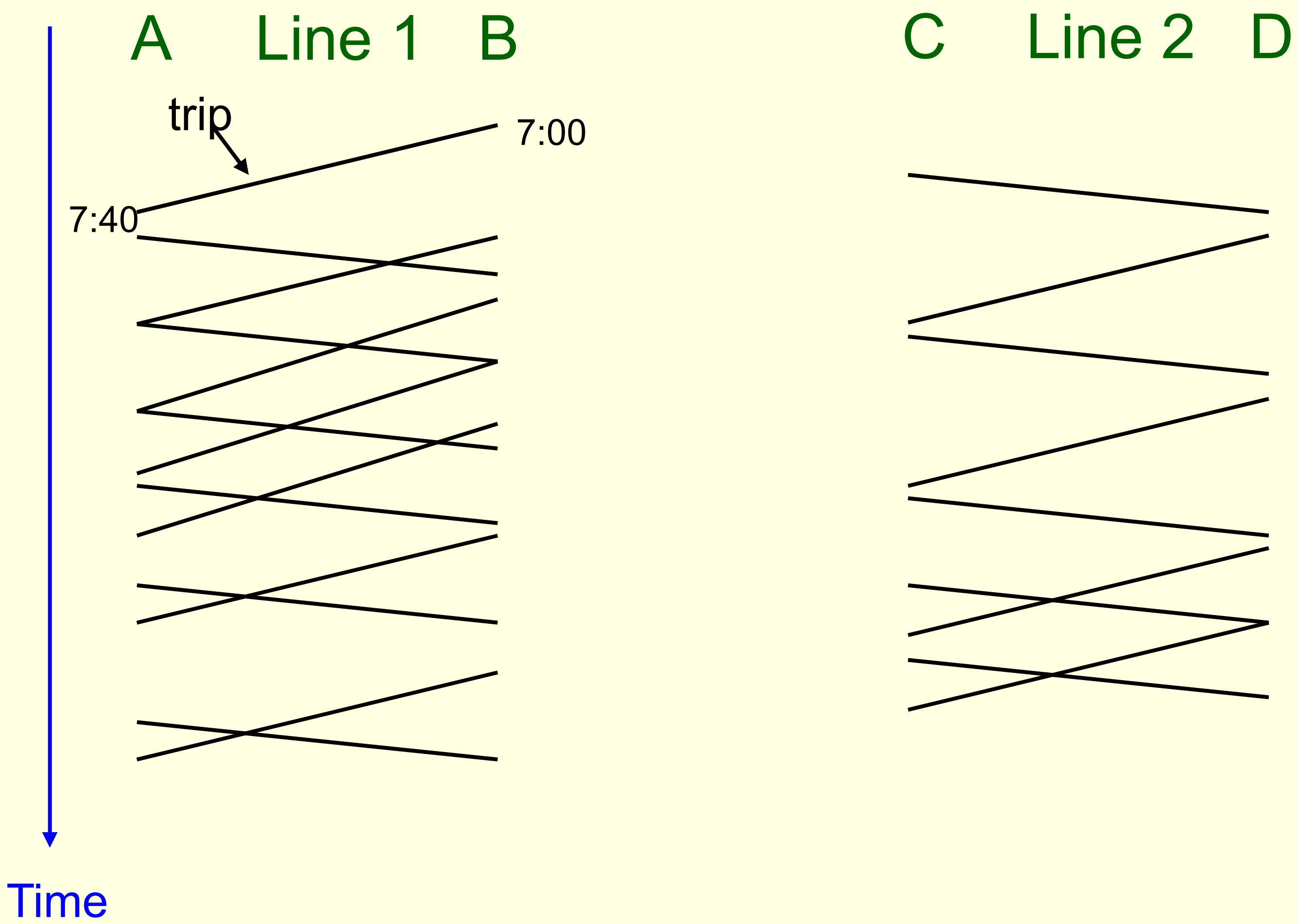
Costs:

- Daily cost of a vehicle (maintanance)
- Cost of the covered distance (transportation costs, deadhead costs)

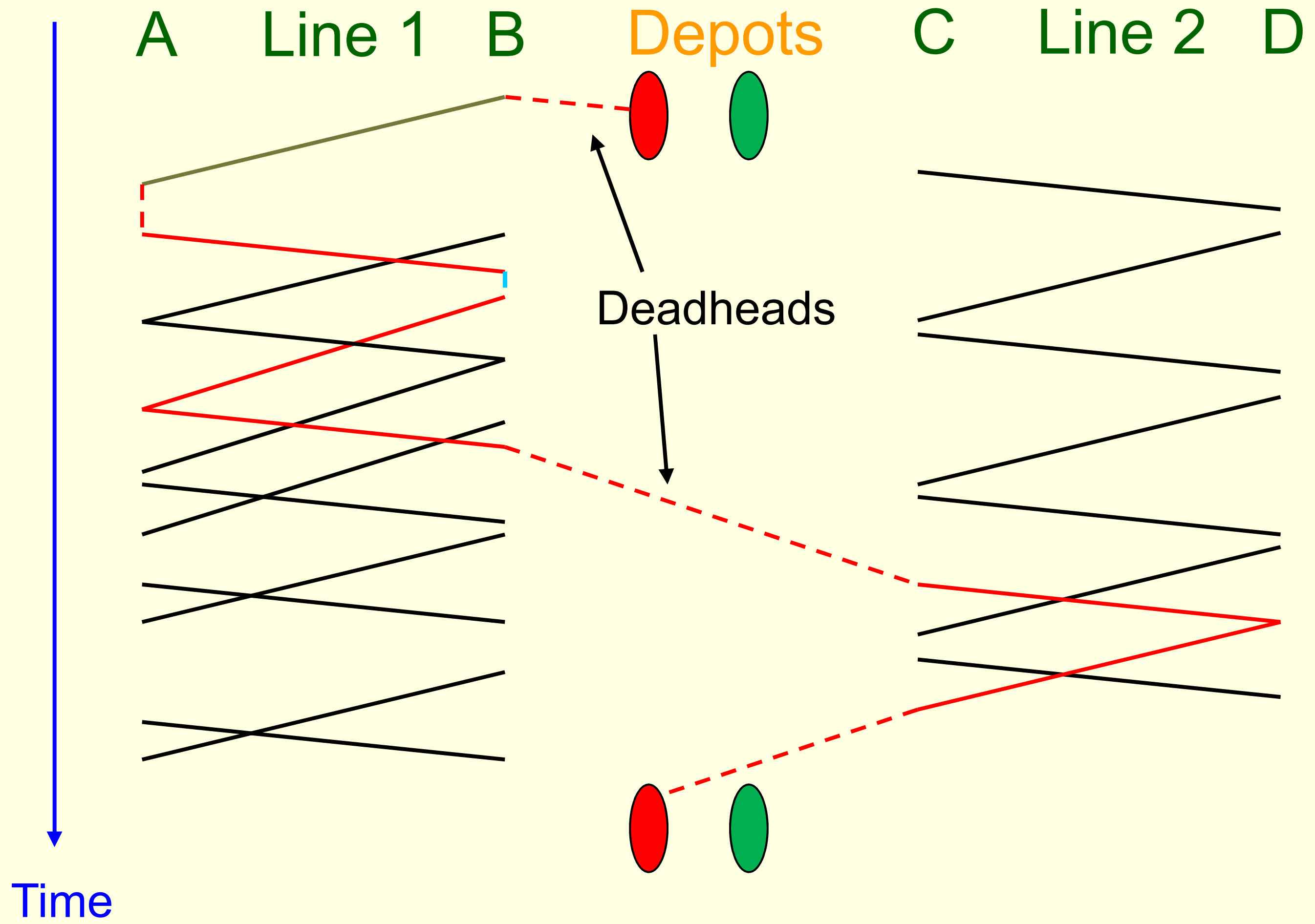
Aim:

- Execute each trip exactly once (with regards to depot compatibility and capacities), minimizing the cost

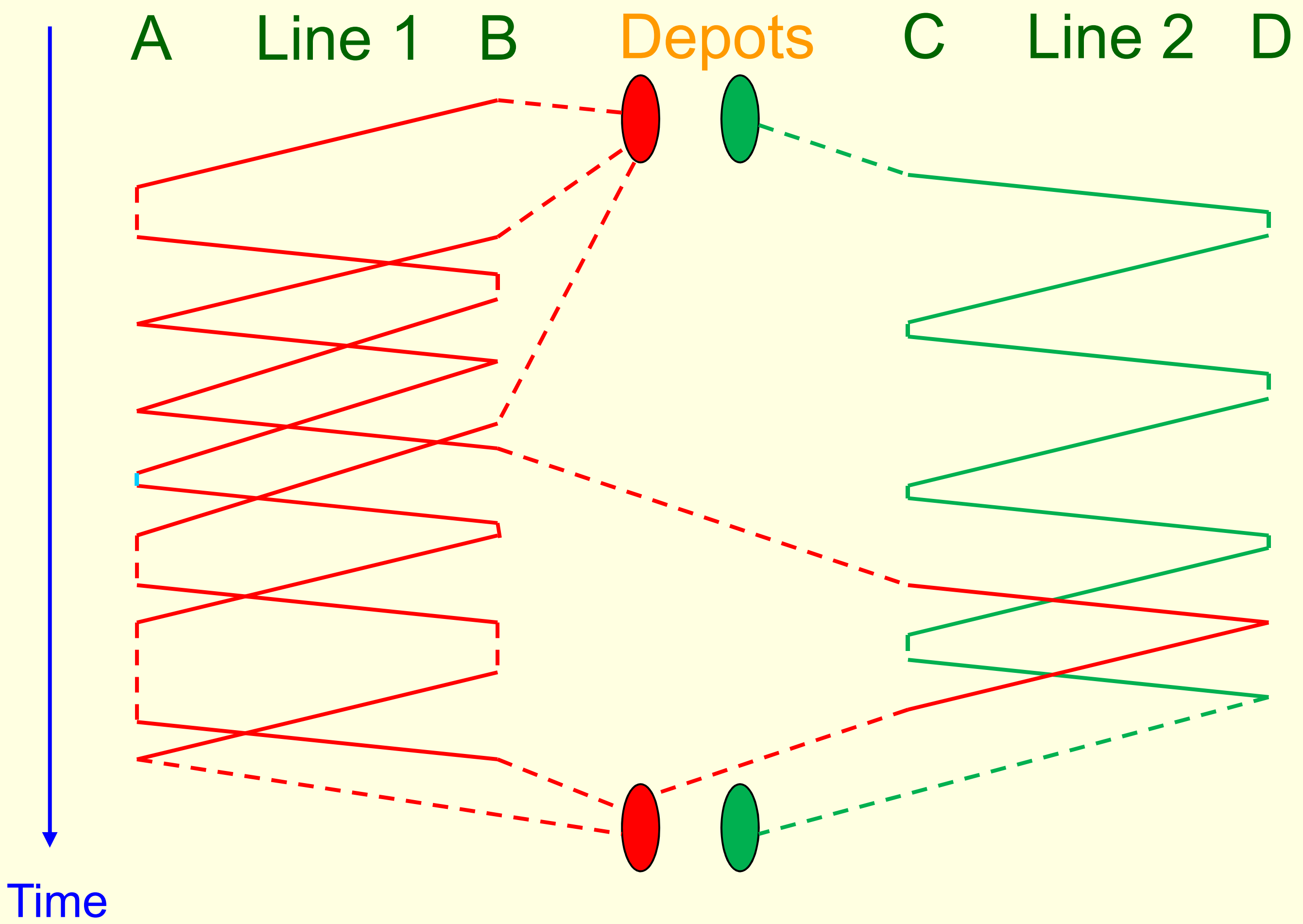
Vehicle scheduling problem



Vehicle scheduling problem



Vehicle scheduling problem

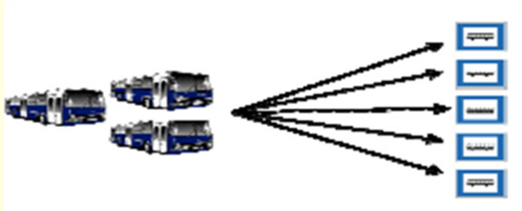


Vehicle scheduling problem

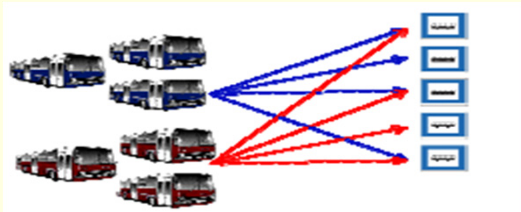
- Single-Depot Vehicle Scheduling Problem(SDVSP)
  - Solvable in polynomial time

  (Matching problem,

  Minimum cost network flow)



- Multiple-Depot Vehicle Scheduling Problem(MDVSP)
  - NP-hard (Bertossi et al., 1987)



Solution by multi-commodity network flow minimization

- Connection based network model (Löbel, 1997)
- Time space network model (Kliewer et al., 2006)

Connection based network

D = set of depots

U = set of trips

V = set of vehicles

$D_u \subseteq D$ set of those depots which can serve the trip $u \in U$

$U_d \subseteq U$ set of those trips which can be served by the depot $d \in D$

$at(d)$ = arrival time to the depot d

$dt(d)$ = departure time from the depot d

$at(u)$ = arrival time of the trip u

$dt(u)$ = departure time of the trip u

Trips u_i and u_j are compatible if $at(u_i) \leq dt(u_j)$ and $dt(u_j) - at(u_i) \geq T_{ij}$

Connection based network
(define a network)

Let N be the set of vertices of the network. Then

$$N = \{\, dt(u) |\, u \in U\} \,\cup \{\, at(u) |\, u \in U\} \cup \{\, dt(d) |\, d \in D\} \cup \{\, at(d) |\, d \in D\}$$

If E_d is the set of timetabled trips belonging to the depot d, then

$$E_d = \{(dt(u), at(u)) |\, u \in U_d\}, \qquad \forall d \in D$$
$$B_d = \{(at(u), dt(v)) |\, u, v \in U_d \text{ are compatible}\}, \forall d \in D$$

where B_d is the set of the depot deadhead trips.

Connection based network

(define a network)

Let R_d be the set of ingoing and outgoing trips of depot d, then

$$R_d = \{ \left(dt(d), dt(u)\right), \left(at(u), at(d)\right) \middle| \, u \in U_d\}, \qquad \forall d \in D$$

Let K_d is the set of depot-circle arcs of the depot d, then

$$K_d = \{(at(d), dt(d)\}, \qquad \forall d \in D$$

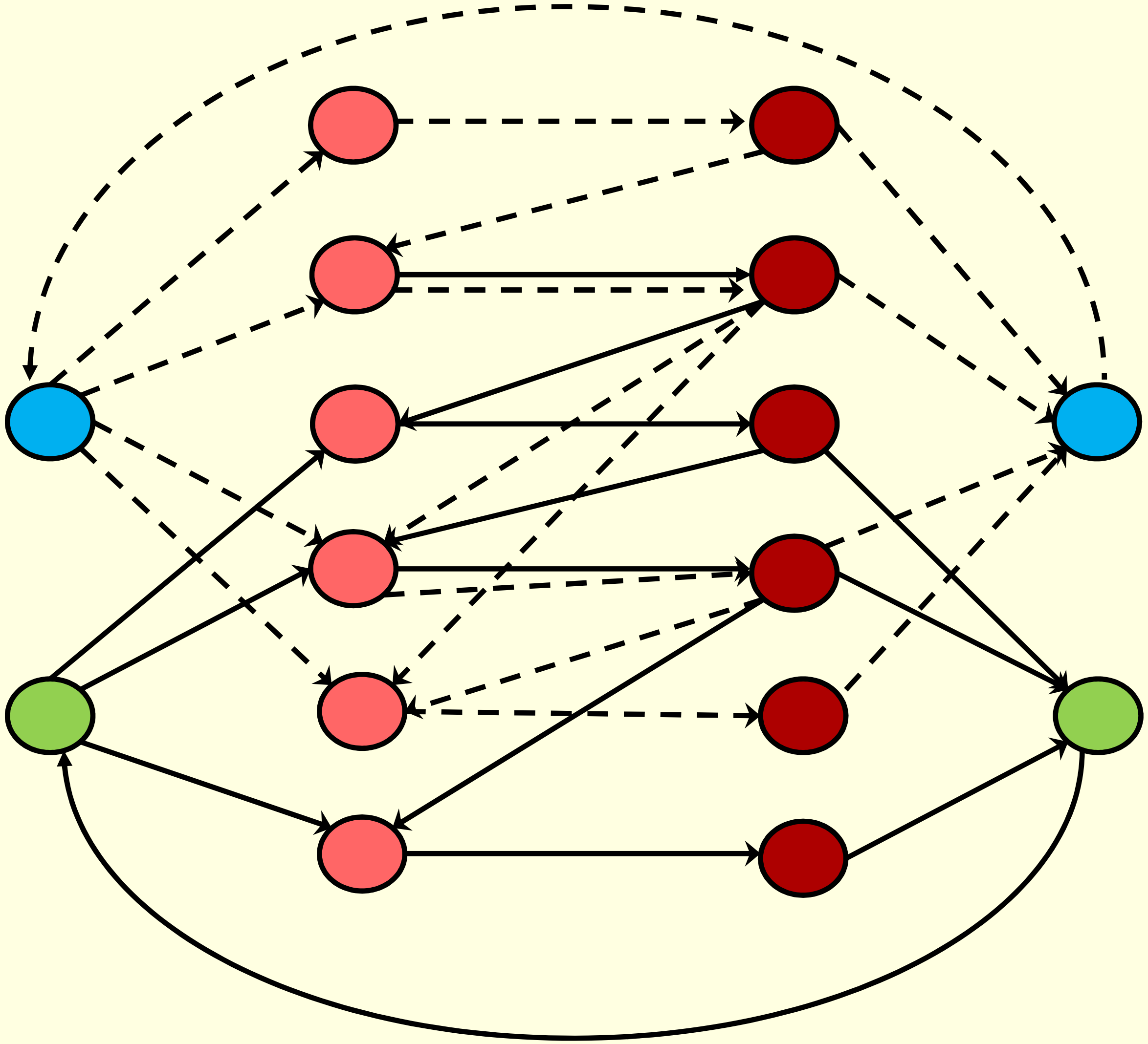
Then the set of arcs belonging to the depot d in the network is

$$A_d = E_d \cup B_d \cup R_d \cup K_d$$

and

$$E = \cup_{d \in D} A_d$$

Connection based network
(define a network)



Connection based network

(define MDVSP on the network)

So, we have a directed graph $G = (N,E)$. We can define the MDVSP on this graph as follows:

Let X be an integer vector with dimension of $|E|$. Its component is belonging to the arc $e \in E$ is x_e^d if $e \in A_d$.
X represent a multicommodity flow.

Considering a schedule S. Then x_e^d we can state

$$x_e^d = \begin{cases} 1, & if \ \ x_e^d \in S \\ 0, & otherwise \end{cases}$$

Connection based network
(define MDVSP on the network)

1. No common edges for two trips:

$$\sum_{d\epsilon Du,\ e=(dt(u),at(u))\epsilon Ed} x_e^d = 1\ \ \forall u\ \epsilon\ U.$$

2. Vehicles must return to the depots:

$$\sum_{e\epsilon n+} x_e^d - \sum_{e\epsilon n-} x_e^d = 0, \forall u\ \epsilon\ U, \forall n\ \epsilon\ N$$

3. If we have capacity constraint for the depots:

$$x_{at(d),dt(d)}^d \leq k_d$$

Connection based network
(define MDVSP on the network)

The objective function is

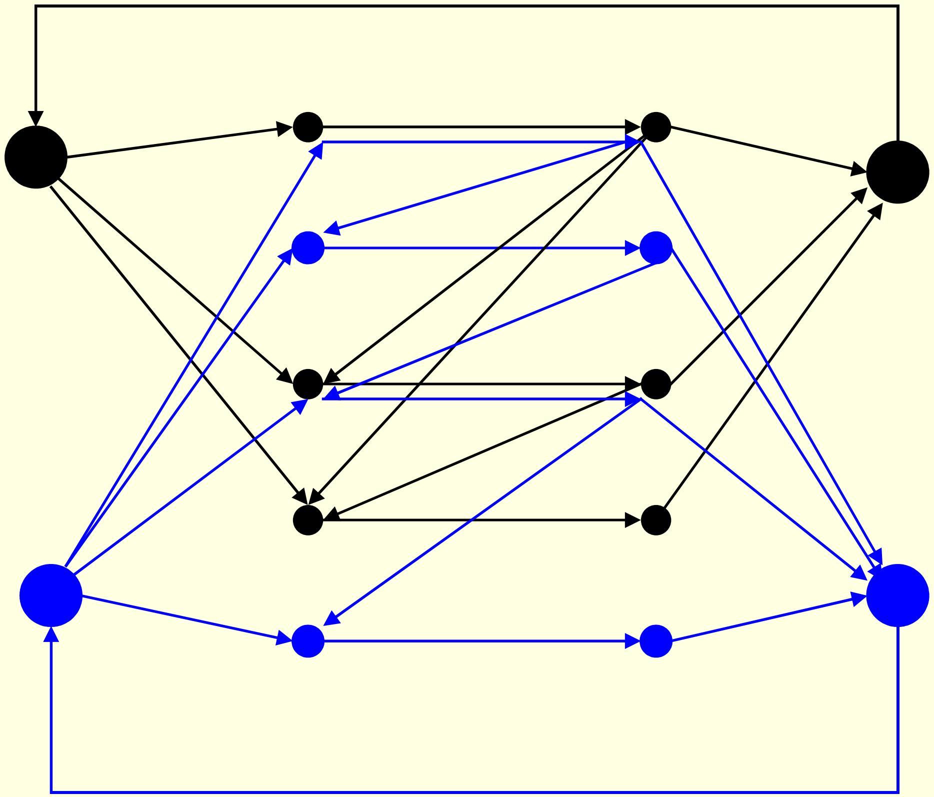
$$min \sum_{e \epsilon E} c_e x_e$$

where c_e is the cost of the edge e.

Connection based network
(define MDVSP on the network)

- Large size
  - Due to high number of possible deadheads

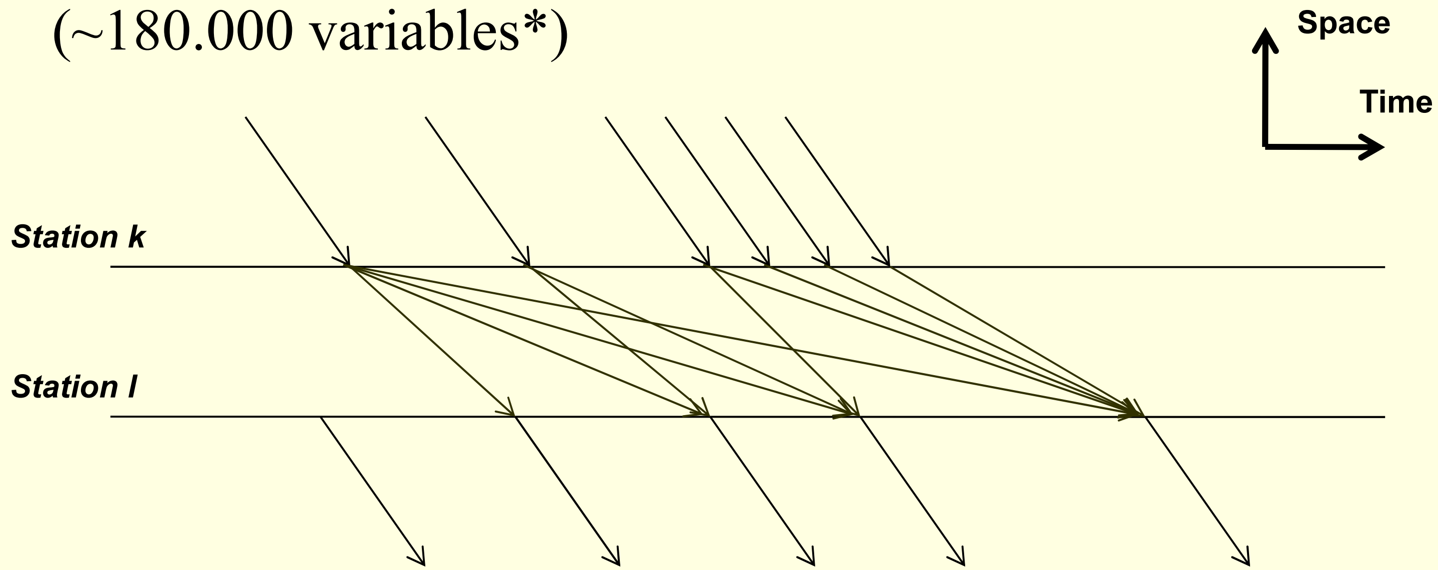
(~5.000.000 variables*)



*Tisza Volán test case

Time space network

- Introduces timelines
- Nodes are points of time
- Aggregates possible connections into waiting edges
  - Significant decrease in model size (~180.000 variables*)



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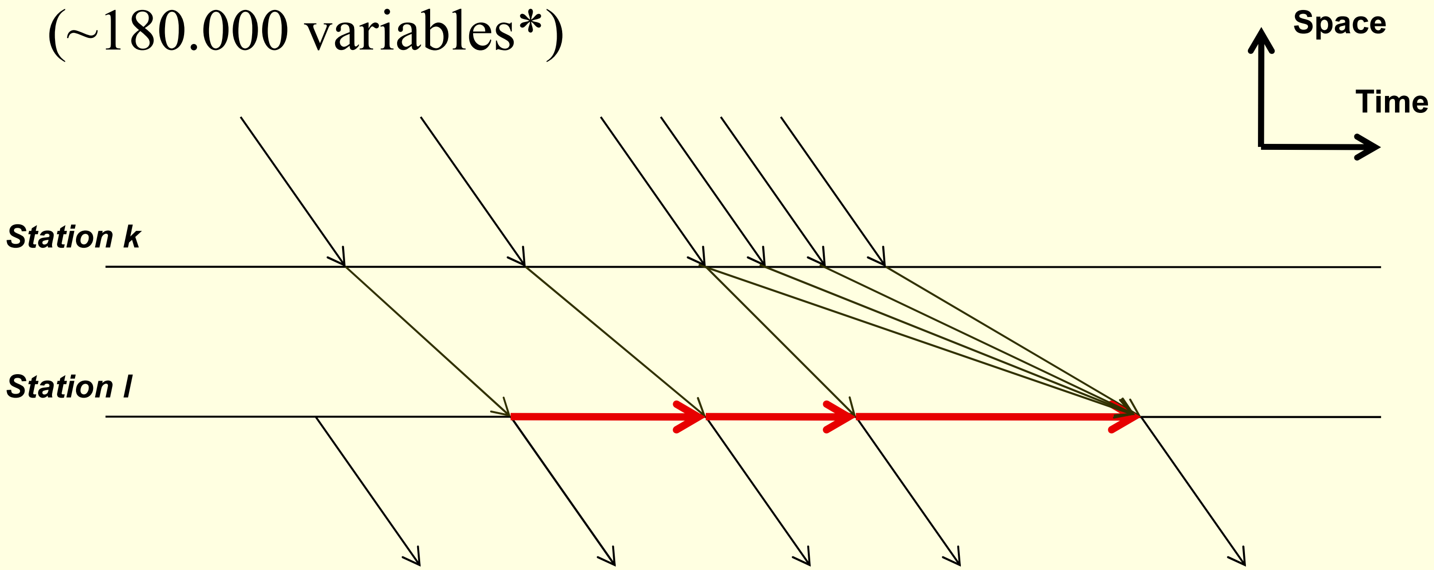

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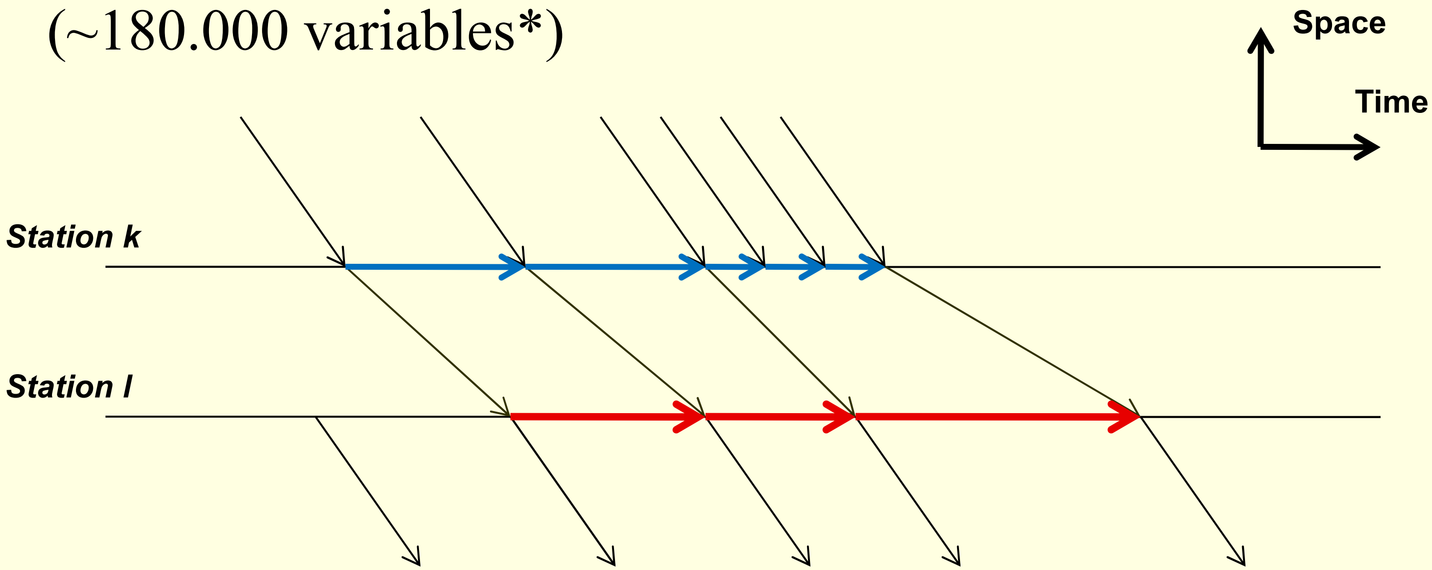

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MDVSP Solution Methods

- Mathematical methods
  - Exact solution of the IP
  - Column generation
  - Lagrangian relaxation
- Combinatorial
  - Tabu search
- Mixed methods
  - Rounding heuristic
  - Variable fixing (Gintner et al., 2005)

Application oriented (greedy) variable fixing
(Dávid & Krész, 2013)

- Based on the idea of Gintner et al.
- An SDVSP relaxation is solved for the problem
- Consecutive trips of the result are fixed together, if they share some common property:
  - Same number of depots (at least one compatible)
  - Belongs to the same bus line (and have a compatible depot)

Case study:
Szeged urban bus transportation

- Fourth largest city of Hungary
- Population: 168.273
- Bus company: Tisza Volán, Urban Transport Division, www.tiszavolan.hu
- 40 lines, 120 buses
- 4 depots: combinations of
  - Conventional vs articulated (gas fuel)
  - Low floor vs normal

Test results

	Time ratio(%)*	„bad” working pieces**	Cost difference max.(%)
Rounding	50,53	3,5	0,1029
Variable-fixing	14,44	13,75	0,2694
Greedy-chains	7,97	4,75	1,0219

•Comparing to „first feasible exact solution”

**not „driver-friendly”

- Effeciency
  - running time: variable fixing and greedy-chains
  - cost: all heuristics perform well
- Integrity with driver rostering
  - a rounding and greedy-chains

⇒ The greedy-chains heuristics satisfies all aspects

Vehicle Assignment Problem

Scheduling problems in public transportation

Vehicle scheduling

Driver scheduling

Driver rostering

Vehicle assignment
(licence plated buses)

Vehicle assignment problem

- Problem of the VSP:
  - Vehicles in the depots are considered uniform (low floor)
  - Hard to integrate vehicle specific tasks to the model
    - refueling, parking, etc.
- Aim: Solving vehicle scheduling with assigning physical buses
- Satisfy vehicle-specific requirements
  - Refueling
  - Parking
  - Maintenance
- Classical MDVSP models do not support these kind of requirements

Vehicle assignment problem

Solution methods

- Sequential approach
  - Transform an initial vehicle schedule with regards to the vehicle-specific tasks
- Integrated model
  - Build a model for the problem that takes these tasks into consideration
  - It is a 3D asignment model:
  - - lines
  - - „phisical” vehicles
  - - vehicle-specific events

Vehicle assignment problem
Sequential approach
(Árgilán et al, 2013)

Input:

- Set of vehicle schedules
  - Solving an MDVSP model
- Set of vehicles
- Refueling stations with parameters
  - Fuel types, capacity of fuel pumps, opening times
- Fuel pumps:
  - Service times with fixed length
  - Service time may vary depending on fuel type

Vehicle assignment problem
Sequential approach

- Variable:

$$X_{ijkt} = \begin{cases} 1 \text{ if schedule } i \text{ is refueled with vehicle } j \text{ at station } k \text{ in time } t \\ 0 \text{ otherwise} \end{cases}$$

- X_{ijkt} exists (not vorbidden), if
  - The depot *i* corresponds to the depot of vehicle *j*
  - The fuel type *k* corresponds to the fuel type of vehicle *j*
  - Schedule *i* is idle in time period *t*
  - The running distances allow the refueling at time *t*
  - Other conditions can be added

Vehicle assignment problem

Sequential approach

$$\min \sum_{i,j,k,t} c_{ijkt} x_{ijkt}$$

s.t.

$$\sum_{j,k,t} x_{ijkt} = 1 \text{ for all } i, (1 \le i \le n)$$

$$\sum_{i,k,t} x_{ijkt} \le 1 \text{ for all } j, (1 \le i \le m)$$

$$\sum_{i,j} x_{ijkt} \le d_{kt} \text{ for all } k, t(1 \le k \le p), t(1 \le t \le T_d(k))$$

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Vehicle assignment problem
Sequential approach

- Solve the multi-dimensional assignment problem above
- Problem: „dense” schedules
  - Change for a different bus
  - Remove events
  - New buses

Vehicle assignment problem

Sequential approach – Test cases

Problem	Trips	Depot	Sched. phase	Assign. phase	B	SB	Decr.
Szeged#1	2724	4	1179	14	107	96	8,28%
Szeged#2	2690	4	872	8	107	96	8,28%
Szeged#3	1981	4	431	5	65	54	14,92%
Szeged#4	1768	4	250	1	54	44	16,52%

Vehicle assignment problem
Integrated approach
(Dávid et al, 2014)

Input:

- Set of trips (T)
- Set of different vehicle types (V)
  - common structural parameters:
  - depot, fuel type, capacity, etc.
- Refueling possibilites (R)
  - All the legal time periods, where a vehicle can be refueled
  - capacity k_r for every $r \epsilon R$

Vehicle assignment problem
Integrated approach

- S_v: Set of legal schedules of vehicle v

$$x_s = \begin{cases} 1 \text{ if schedule } s \text{ is carried out} \\ 0 \text{ otherwise} \end{cases}$$

- Variable to connect tasks and schedules

$$a_{is} = \begin{cases} 1 \text{ if task a is carried out in schedule } s \\ 0 \text{ otherwise} \end{cases}$$

- (tasks can be trips, refuelings, or others)

Vehicle assignment problem
Integrated approach

$$\sum_{v=1}^{n} \sum_{s \in S_v} c_s x_s \to min$$

s.t.

$$\sum_{v=1}^{n} \sum_{s \in S_v} a_{ts} x_s = 1 \qquad \forall t \in T$$

$$\sum_{v=1}^{n} \sum_{s \in S_v} a_{rs} x_s \leq k_r \qquad \forall r \in R$$

$$\sum_{s \in S_v} x_s \leq q_v \qquad \forall v \in V$$

$$x_s \in \{0, 1\} \qquad \forall s \in S_v$$

Vehicle assignment problem
Integrated approach

- Solve the problem using column generation
  - Master problem
  - Pricing problem
    - Resource constrained shortest path
    - Check refueling capacities

Vehicle assignment problem

Integrated approach – Test cases

Problem	Trips	Depots	Integrated step	CB	IB	Decr.
Szeged#1	2724	4	4264	107	95	13,34%
Szeged#2	2690	4	3542	107	96	14,22%
Szeged#3	1981	4	2745	65	54	16,04%
Szeged#4	1768	4	2687	54	44	19,63%

„Driver-friendly” Vehicle Scheduling

„Driver-friendly" vehicle scheduling
(Árgilán et al., 2011)

- Schedules given by vehicle scheduling/assignment
- The problem of „dense" schedules
  - Driver rules have to be considered
- Sequential heuristic
  - Uses the results of the previous phases

„Driver-friendly" vehicle schedules

Classification

Class A:  length ≤ M, needs 1 driver

Class B:  M < length ≤ 1,5*M, needs 2 drivers (1,5 drivers actually)

Class C:  1,5*M < length ≤ 2M, needs 2 drivers

„Driver-friendly” vehicle schedules
Transformation

- Cut-and-join for trips of class B
- Split trips of class C
- Different number of break depending on length
- Insertion of breaks
  - No transformation needed
  - Trips have to be removed
    - Can be moved to another schedule
    - Have to be moved to the „free-list”

„Driver-friendly" vehicle schedules

Iteration steps

- New input for vehicle scheduling
  - „Free-list"
- Classification and Transformation on the new schedules
- Do while the „free-list" is empty

„Driver-friendly” vehicle scheduling