The Stable Set Polytope of "Almost" All Claw Free Graphs

Paolo Ventura

IASI-CNR

JOINT WORK WITH: A. Galluccio and C. Gentile

Aussois - 2009

The Stable Set Problem

Given a graph $G = (V, E)$

- A stable set S is a subset of V s.t. if $u, v \in S \Rightarrow \{u, v\} \notin E$.
- Given $w \in \mathbb{Q}_+^V$, the Stable Set Problem consists in finding a stable set of G of maximum weight.
- The Stable Set Polytope, denoted by $STAB(G)$, is the convex hull of the incidence vectors of the stable sets of G.
- The Stable Set problem is NP-hard

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The Stable Set Problem on Claw-Free Graphs

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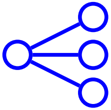

- The Stable Set Problem is polynomial time solvable on claw-free graphs
  - ⇒ Minty 81
  - ⇒ (Tamura and Nakamura 02, Schrijver 03)
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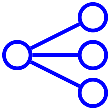

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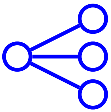

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The Structure of Claw-Free Graphs (Chudnovsky & Seymour ’04)

Connected
claw-free graphs
with no 1-joins

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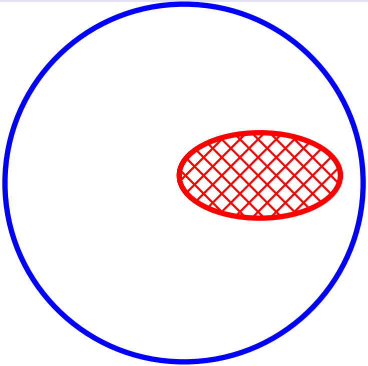

- $\alpha \leq 2$, Clique neughborhood ineq.es (Cook ’87)
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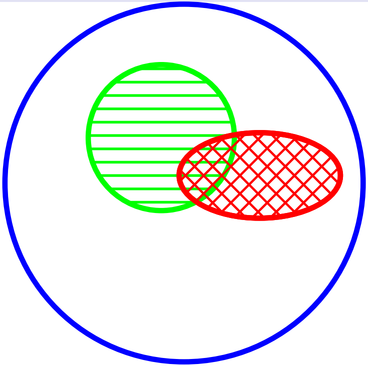

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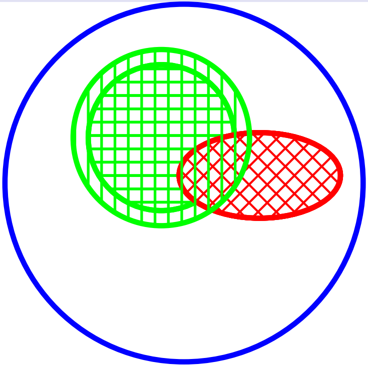

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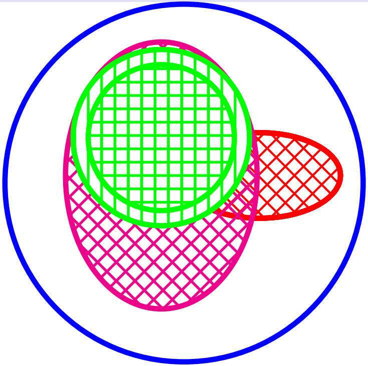

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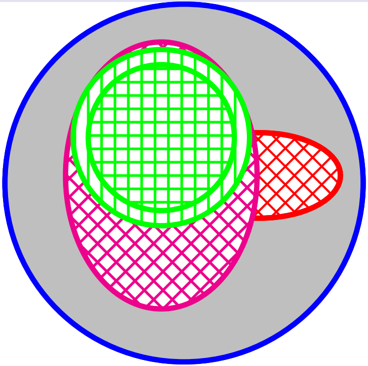

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Fuzzy linear int. strips + Fuzzy XX-strips + Fuzzy antihat strips

? STAB(G) ?

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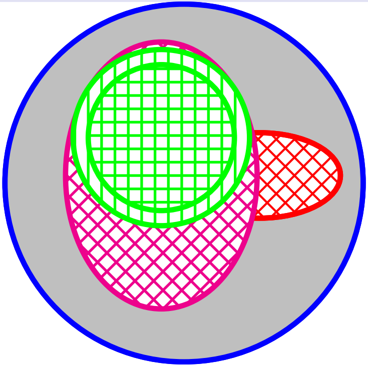

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Strips and
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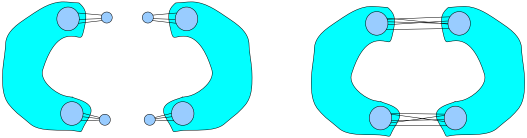

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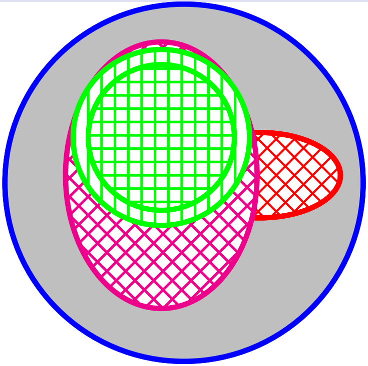

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$\Rightarrow$ Fuzzy linear interval strips + Fuzzy XX-strips + Fuzzy antihat strips

with no homogeneus pairs of cliques $\Rightarrow$ Linear int. strips + XX-strips + Antihat strips

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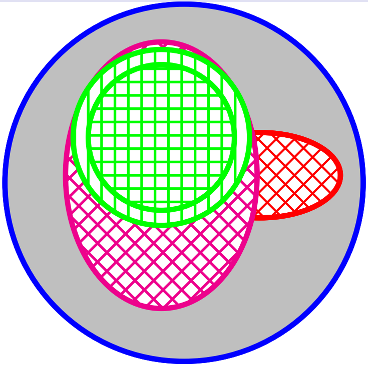

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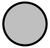
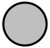
 $\Rightarrow$ Fuzzy linear interval strips + Fuzzy XX-strips + Fuzzy antihat strips

 with no homogeneus pairs of cliques $\Rightarrow$ Linear int. strips + XX-strips + Antihat strips

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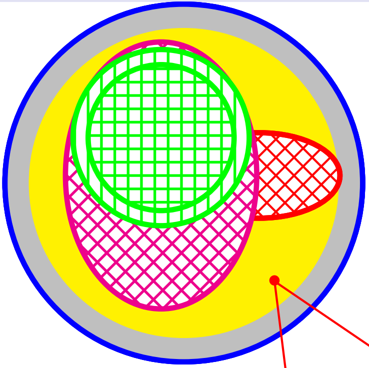

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XX strips

An XX-strip
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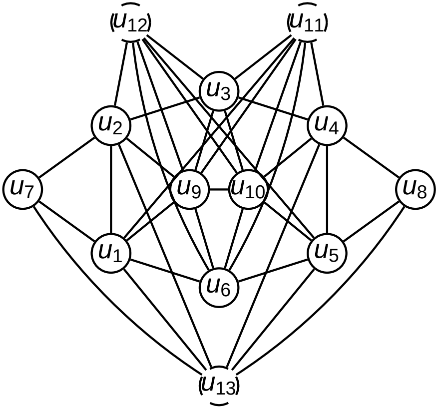

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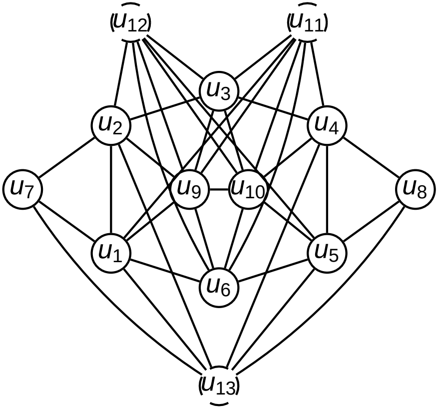
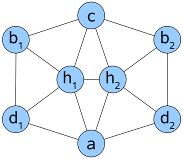


An XX-graph is a composition of fuzzy linear interval strips and XX-strips.

Gears and gear composition

A gear is the graph:



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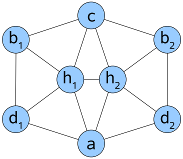

- An edge $\{v_1, v_2\}$ of a given graph $H = (V_H, E_H)$ is said to be simplicial if $K_1 = N(v_1) \setminus \{v_2\}$ and $K_2 = N(v_2) \setminus \{v_1\}$ are two nonempty cliques of H

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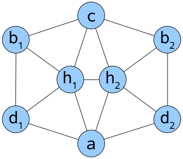

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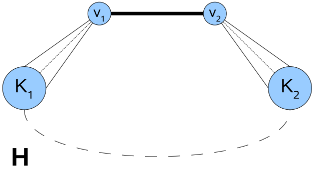
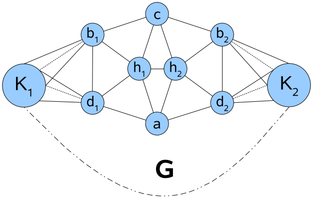
- The composition of a graph $H = (V_H, E_H)$ and a gear B

along the simplicial edge $\{v_1, v_2\} \in E_H$ is the following geared graph $G(H, B, e)$:





G-extendable and geared inequalities

Let H be a graph with a simplicial edge $e = v_1v_2$.

- An inequality (π, π_0) which is valid for $STAB(H)$ is said to be g-extendable (with respect to e) if $\pi_{v_1} = \pi_{v_2} = \lambda > 0$ and it is not the inequality $x_{v_1} + x_{v_2} \leq 1$.
- Let $B = (V_B, E_B)$ be a gear and (π, π_0) be g-extendable with respect to e. Then the inequalities
  - $$\sum_{i \in V_H \setminus \{v_1, v_2\}} \pi_i x_i + \lambda \sum_{i \in V_B \setminus \{h_1, h_2\}} x_i + 2\lambda(x_{h_1} + x_{h_2}) \leq \pi_0 + 2\lambda$$
  - $$\sum_{i \in V_H \setminus \{v_1, v_2\}} \pi_i x_i + \lambda \sum_{i \in V_B \setminus A} x_i \leq \pi_0 + \lambda$$

    where $A \in \{\{b_1, c\}, \{b_2, c\}, \{d_1, a\}, \{d_2, a\}, \{a, c\}\}$

  are called geared inequalities generated by (π, π_0)

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G-liftable and g-lifted inequalities

Let H^e be the graph obtained from H by subdividing the simplicial edge e with a new node t.

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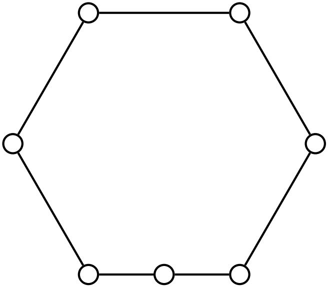
The stable set polytope of a geared graph

Theorem Let $G = (H, B_Y, e)$ be a geared graph. Then the stable set polytope $STAB(G)$ is described by the following linear inequalities:

- clique-inequalities,
- (lifted) 5-wheel inequalities,
- geared inequalities associated with g-extendable facet defining inequalities of $STAB(H)$,
- g-lifted inequalities associated with g-liftable facet defining inequalities of $STAB(H^e)$,
- facet defining inequalities of $STAB(H)$,

Geared inequalities



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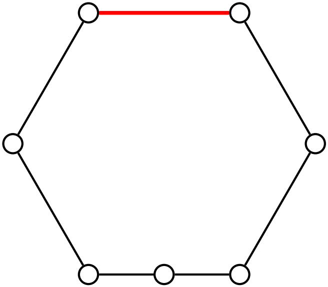

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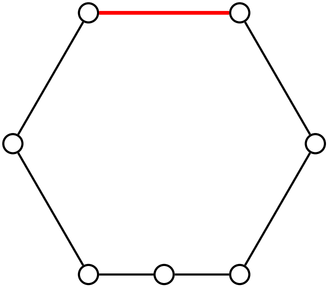

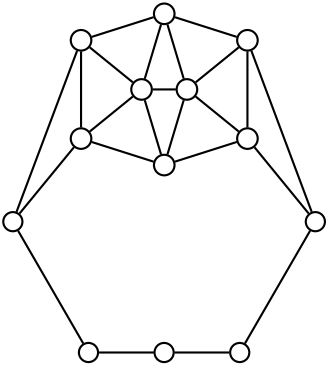

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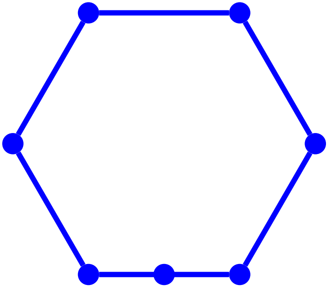

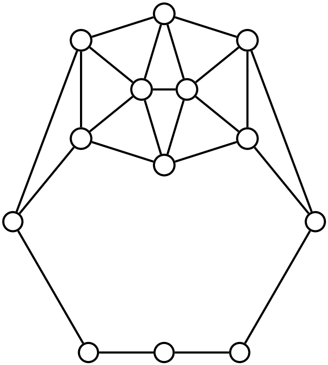

$$\sum_{v \in \bullet} x_v \leq 3$$

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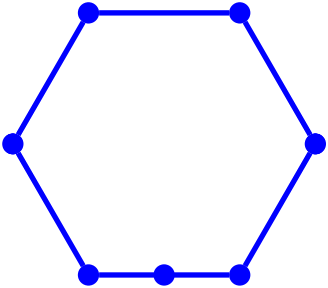

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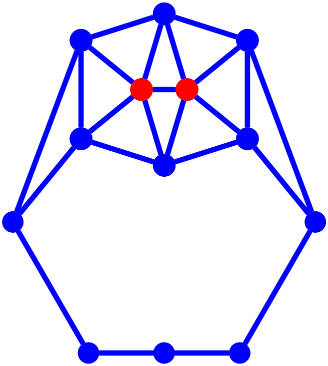

$$\sum_{v \in \bullet} x_v \leq 3 + 2$$

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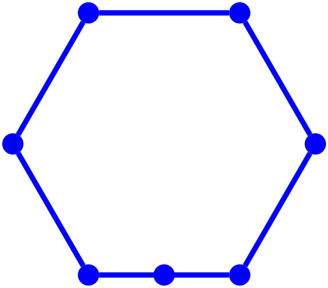

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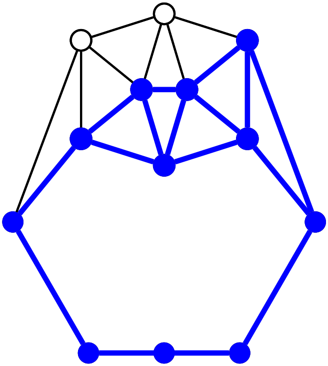

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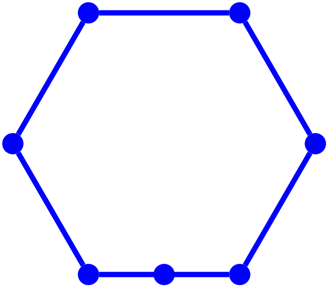

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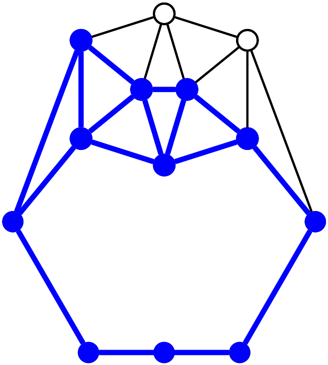

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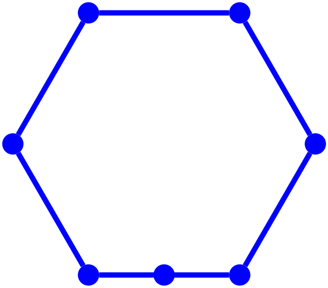

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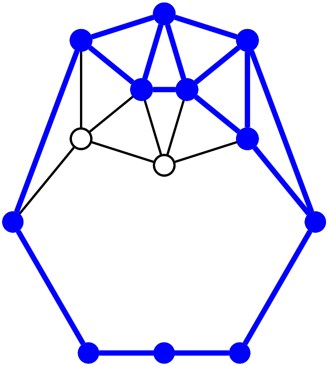
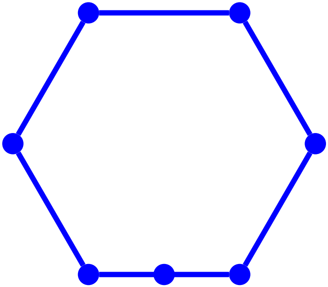
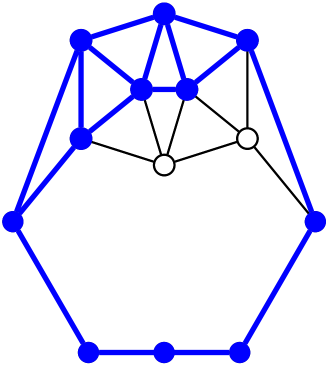


$$\sum_{v \in \bullet} x_v \leq 3 + 1$$

Geared inequalities

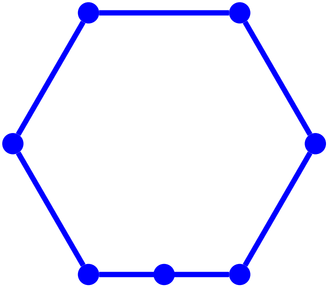


$$\sum_{v \in \bullet} x_v \leq 3$$

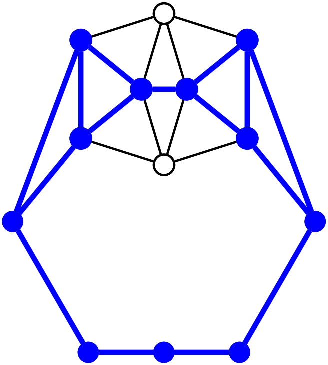


$$\sum_{v \in \bullet} x_v \leq 3 + 1$$

Geared inequalities

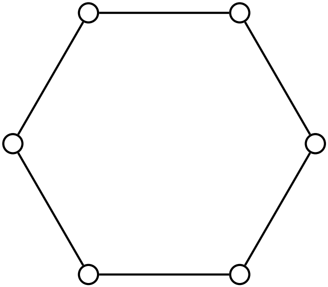


$$\sum_{v \in \bullet} x_v \leq 3$$

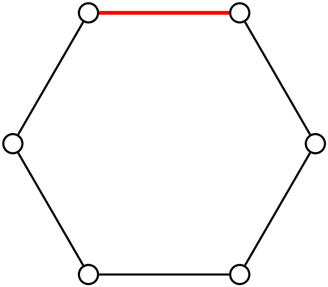


$$\sum_{v \in \bullet} x_v \leq 3 + 1$$

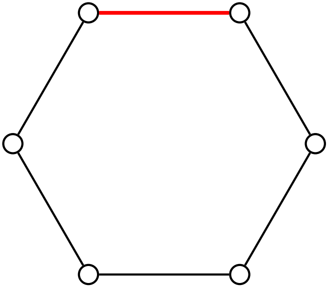
G-lifted inequalities

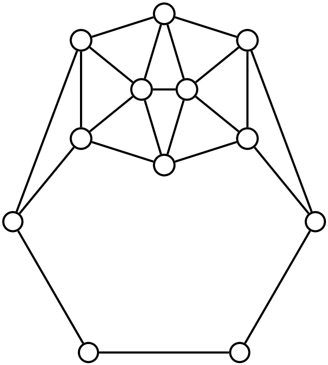


G-lifted inequalities

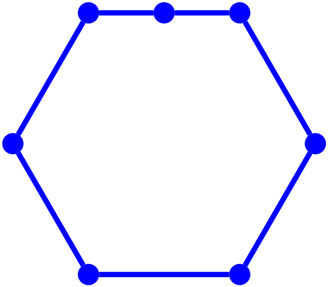


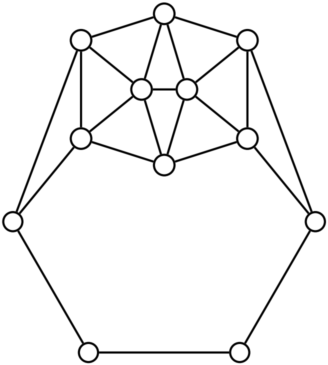
G-lifted inequalities





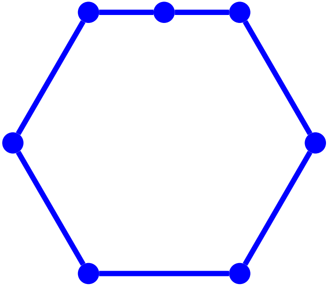
G-lifted inequalities



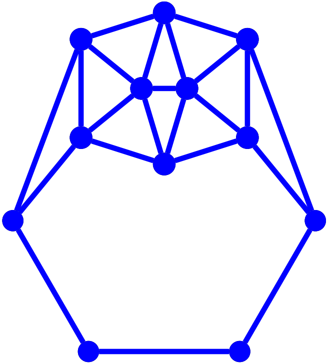


$$\sum_{v \in \bullet} x_v \leq 3$$

G-lifted inequalities

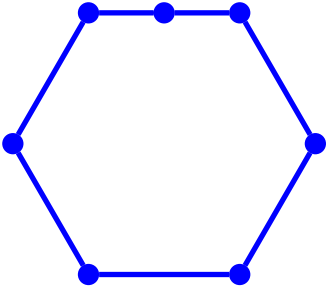


$$\sum_{v \in \bullet} x_v \leq 3$$

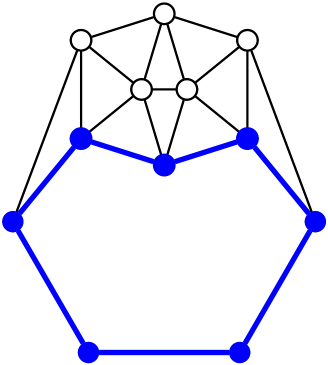


$$\sum_{v \in \bullet} x_v \leq 3 + 1$$

G-lifted inequalities



$$\sum_{v \in \bullet} x_v \leq 3$$



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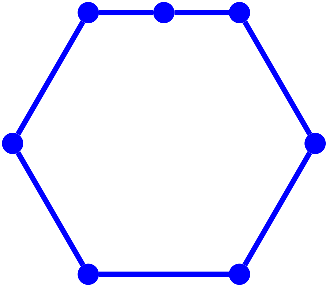
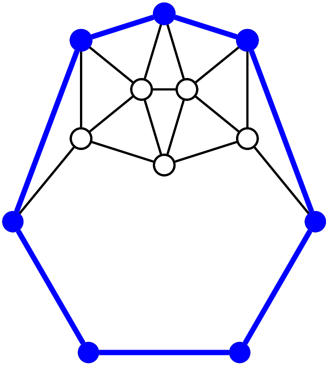
G-lifted inequalities

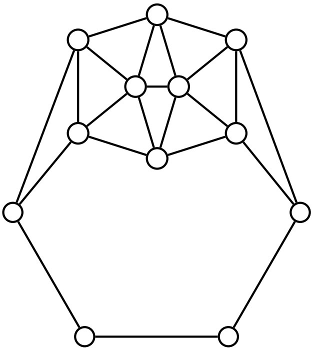


$$\sum_{v \in \bullet} x_v \leq 3$$

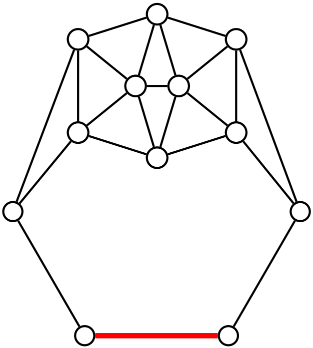


$$\sum_{v \in \bullet} x_v \leq 3$$

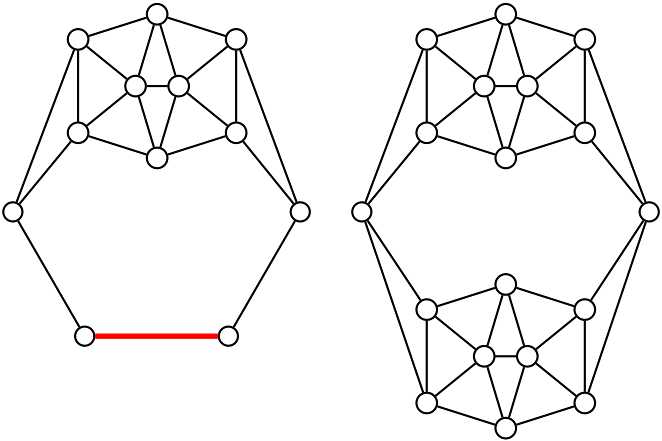
Geared geared graphs



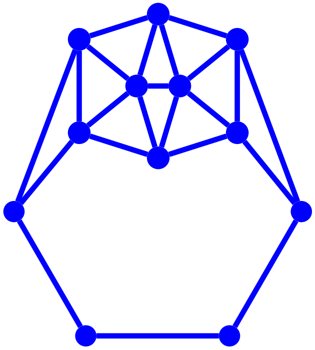
Geared geared graphs



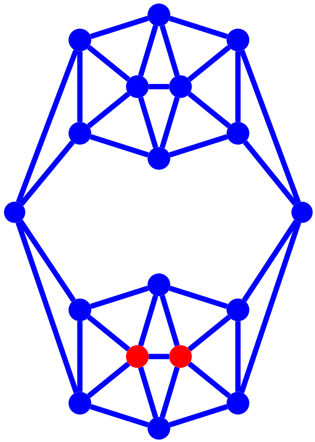
Geared geared graphs



Geared geared graphs

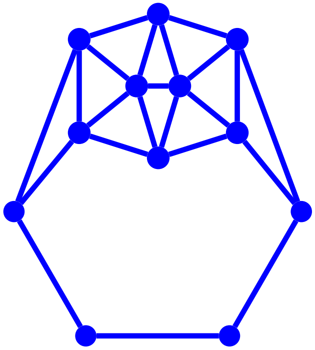


$$\sum_{v \in \bullet} x_v \leq 4$$

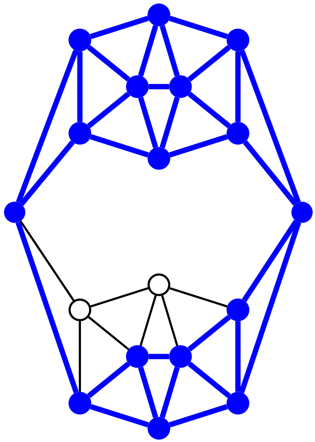


$$\sum_{v \in \bullet} x_v + 2 \sum_{v \in \bullet} x_v \leq 4 + 2$$

Geared geared graphs

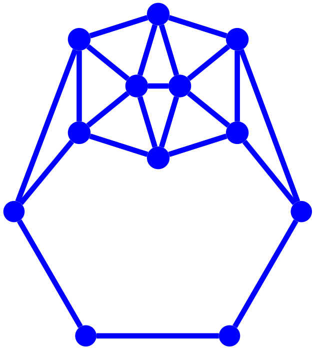


$$\sum_{v \in \bullet} x_v \leq 4$$



$$\sum_{v \in \bullet} x_v \leq 4 + 1$$

Geared geared graphs



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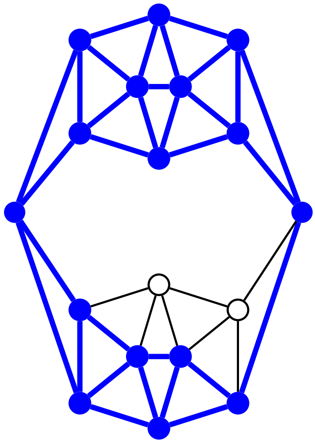
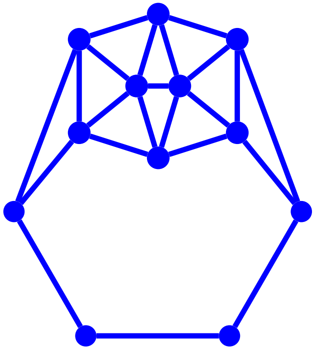


$$\sum_{v \in \bullet} x_v \leq 4 + 1$$

Geared geared graphs



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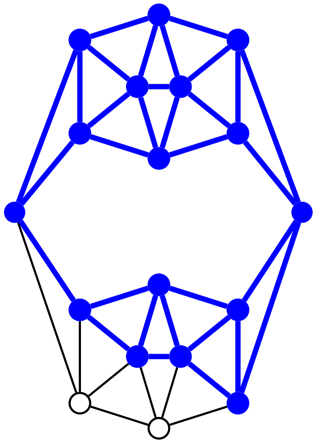
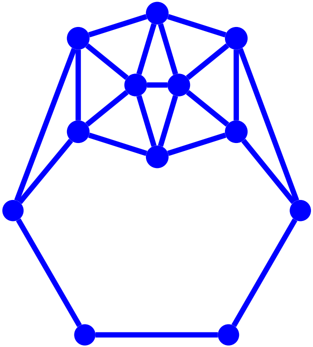
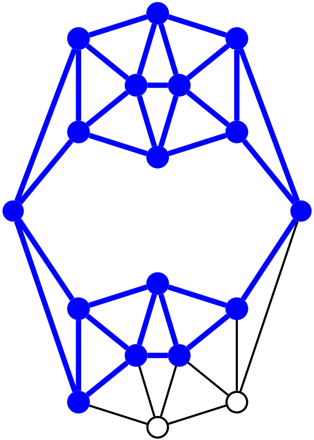


$$\sum_{v \in \bullet} x_v \leq 4 + 1$$

Geared geared graphs

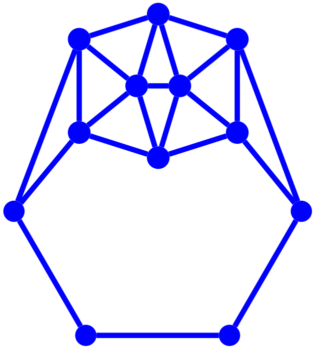


$$\sum_{v\in\bullet} x_v \leq 4$$

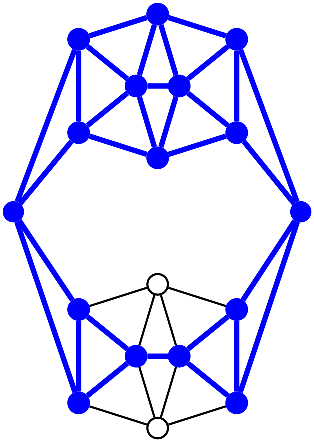


$$\sum_{v\in\bullet} x_v \leq 4+1$$

Geared geared graphs

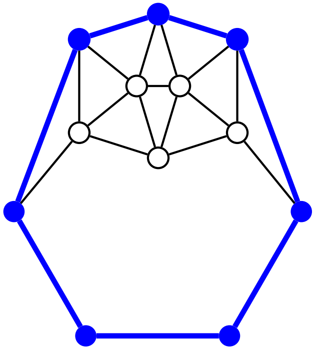


$$\sum_{v \in \bullet} x_v \leq 4$$

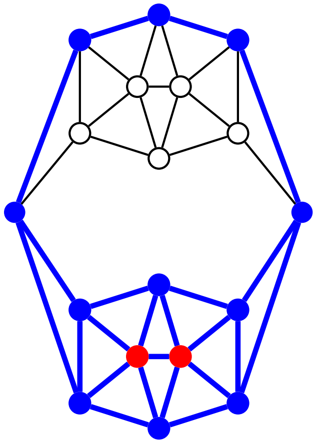


$$\sum_{v \in \bullet} x_v \leq 4 + 1$$

Geared geared graphs

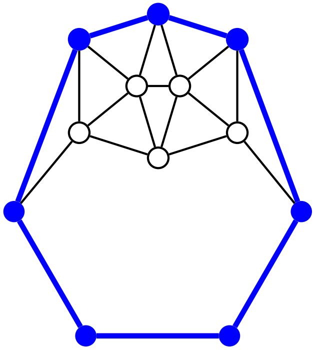


$$\sum_{v \in \bullet} x_v \leq 3$$

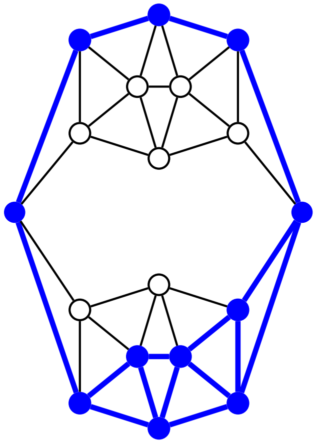


$$\sum_{v \in \bullet} x_v + 2 \sum_{v \in \bullet} x_v \leq 3 + 2$$

Geared geared graphs



$$\sum_{v \in \bullet} x_v \leq 3$$



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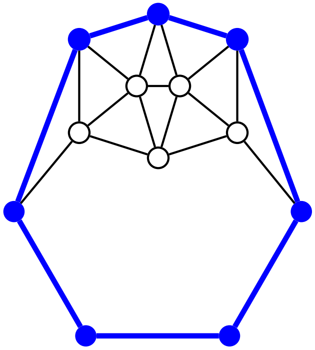
Geared geared graphs



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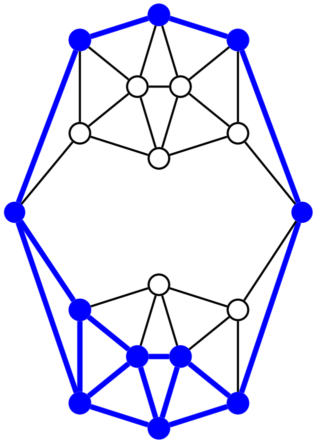

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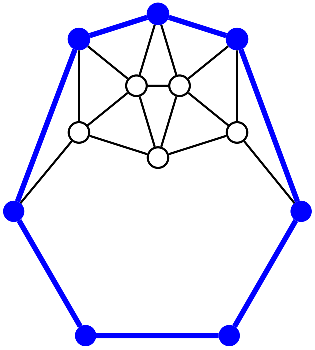
Geared geared graphs



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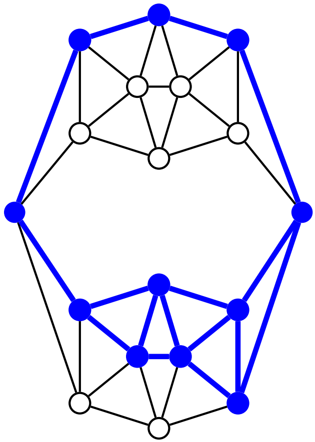

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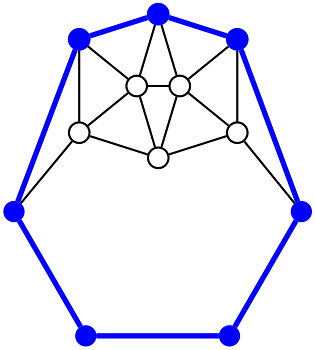
Geared geared graphs



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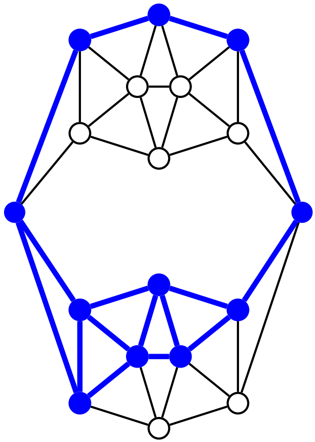

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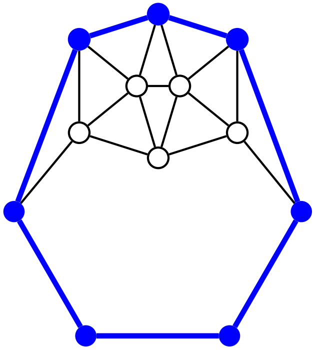
Geared geared graphs



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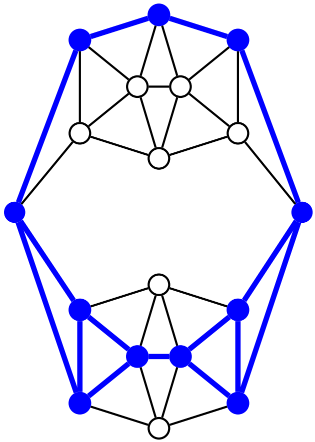
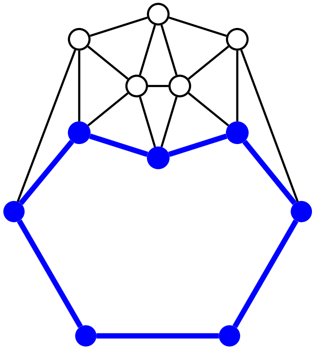
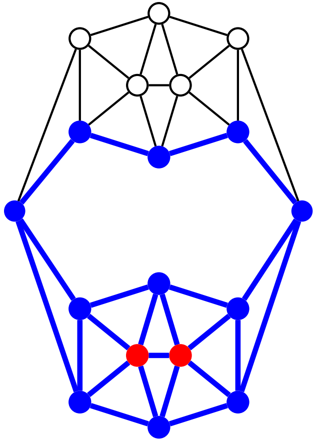


$$\sum_{v \in \bullet} x_v \leq 3 + 1$$

Geared geared graphs

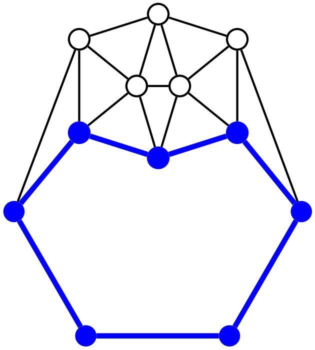


$$\sum_{v \in {\color{blue}\bullet}} x_v \leq 3$$

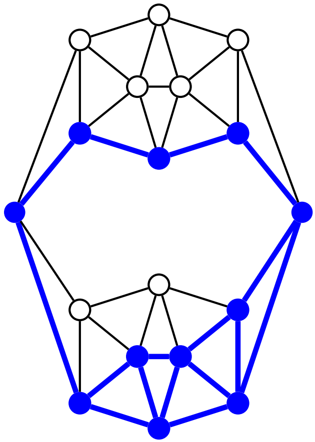


$$\sum_{v \in {\color{blue}\bullet}} x_v + 2 \sum_{v \in {\color{red}\bullet}} x_v \leq 3 + 2$$

Geared geared graphs

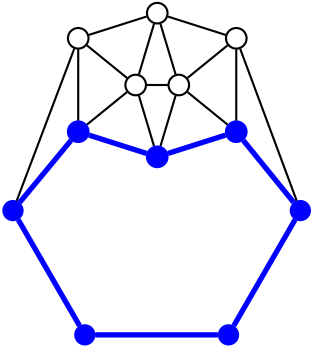


$$\sum_{v \in \bullet} x_v \leq 3$$

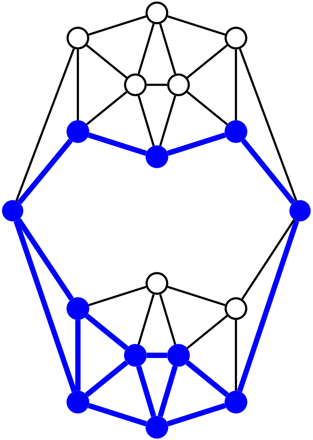


$$\sum_{v \in \bullet} x_v \leq 3 + 1$$

Geared geared graphs

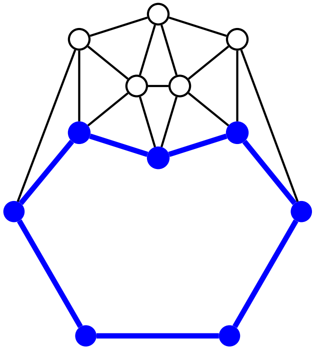


$$\sum_{v\in\bullet} x_v \leq 3$$



$$\sum_{v\in\bullet} x_v \leq 3+1$$

Geared geared graphs



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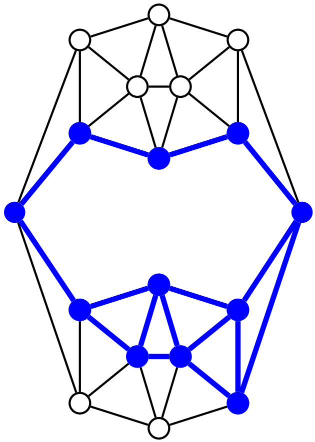

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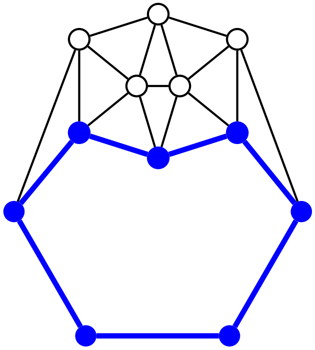
Geared geared graphs



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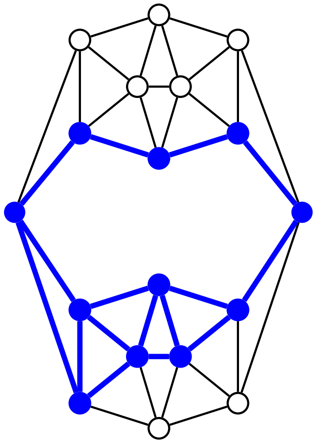

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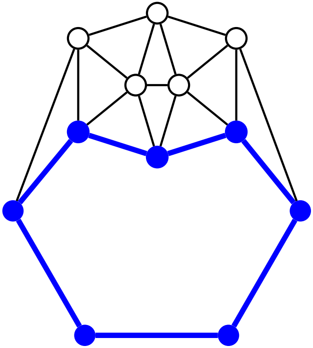
Geared geared graphs



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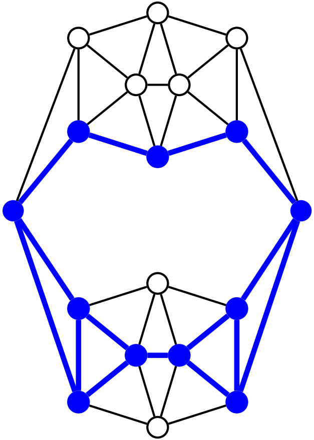
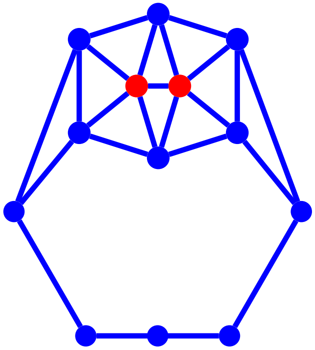
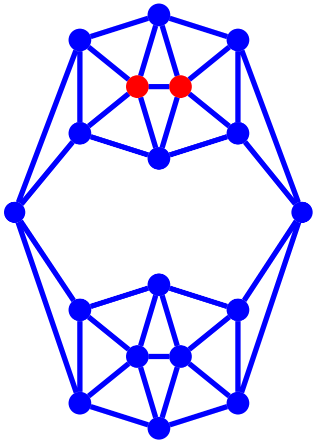


$$\sum_{v \in \bullet} x_v \leq 3 + 1$$

Geared geared graphs

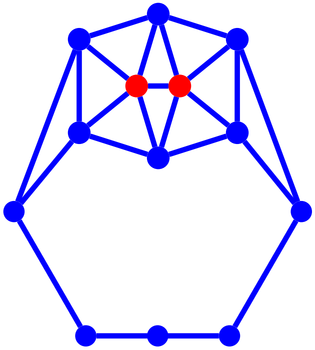


$$\sum_{v \in {\color{blue}\bullet}} x_v + 2 \sum_{v \in {\color{red}\bullet}} x_v \leq 5$$

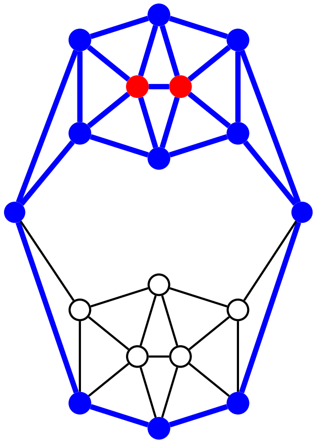


$$\sum_{v \in {\color{blue}\bullet}} x_v + 2 \sum_{v \in {\color{red}\bullet}} x_v \leq 5 + 1$$

Geared geared graphs

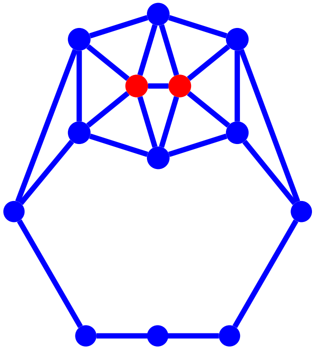


$$\sum_{v \in \textcolor{blue}{\bullet}} x_v + 2 \sum_{v \in \textcolor{red}{\bullet}} x_v \leq 5$$

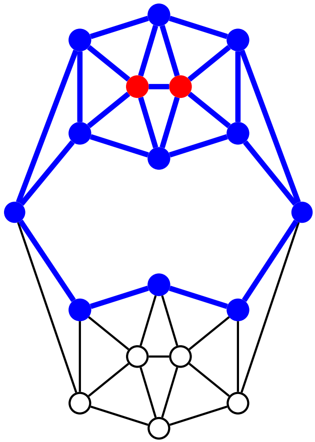


$$\sum_{v \in \textcolor{blue}{\bullet}} x_v + 2 \sum_{v \in \textcolor{red}{\bullet}} x_v \leq 5$$

Geared geared graphs

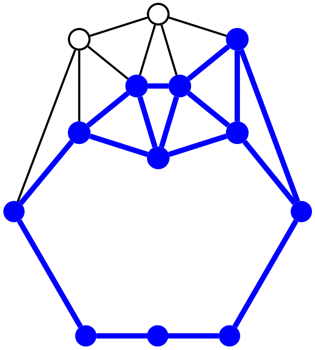


$$\sum_{v \in {\color{blue}\bullet}} x_v + 2 \sum_{v \in {\color{red}\bullet}} x_v \leq 5$$

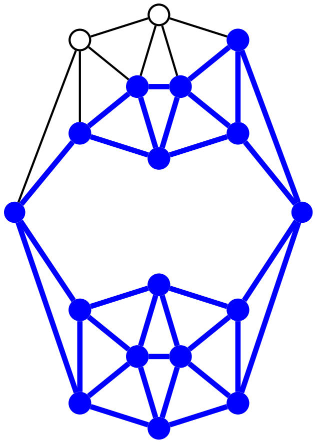


$$\sum_{v \in {\color{blue}\bullet}} x_v + 2 \sum_{v \in {\color{red}\bullet}} x_v \leq 5$$

Geared geared graphs

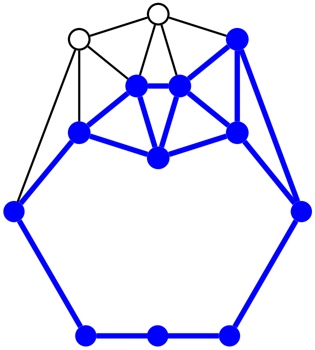


$$\sum_{v \in \bullet} x_v \leq 5$$



$$\sum_{v \in \bullet} x_v \leq 5 + 1$$

Geared geared graphs



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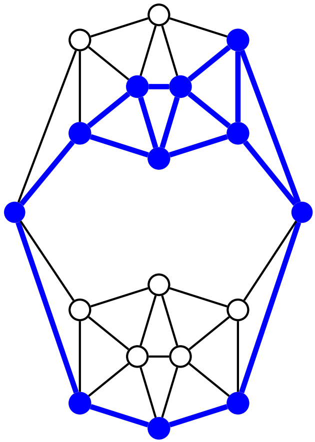

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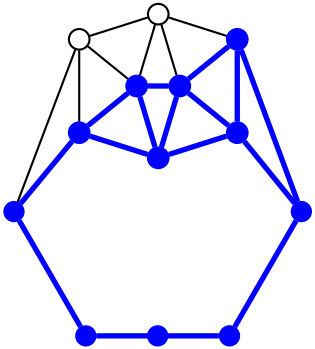

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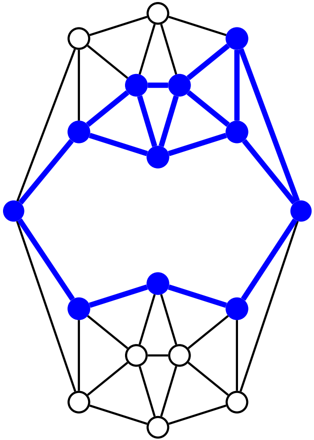

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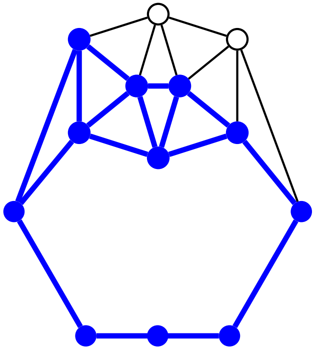

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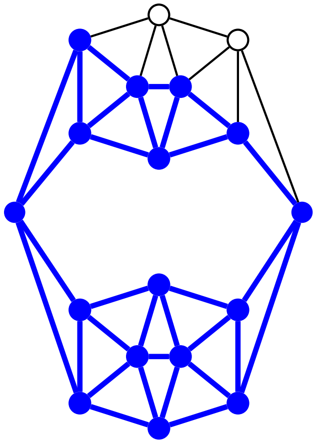

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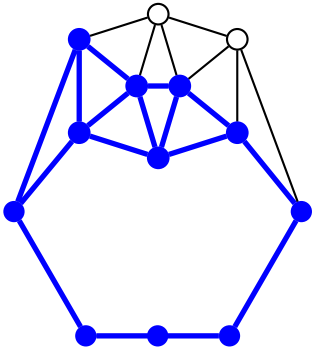

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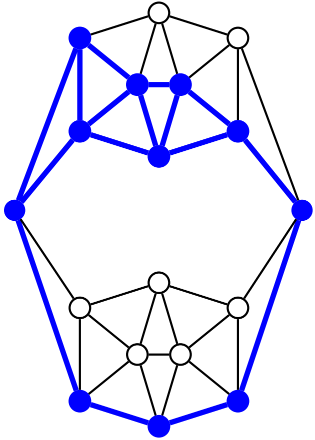

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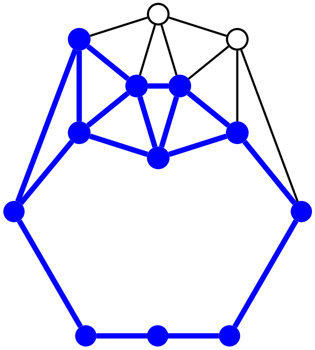

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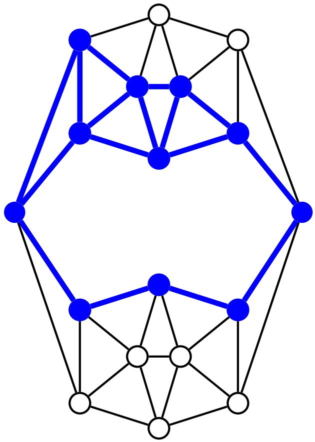

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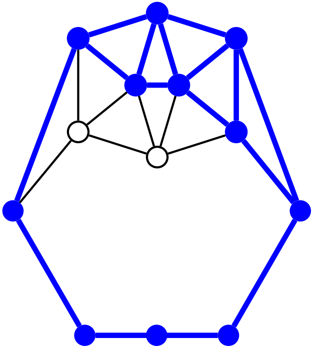

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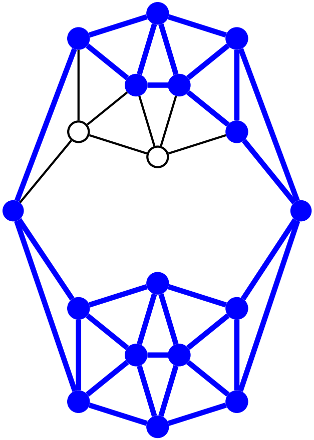

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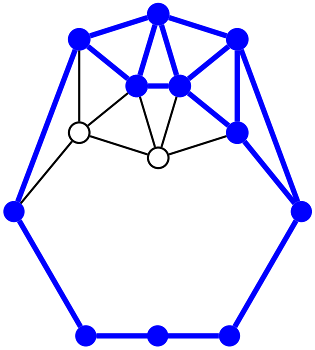

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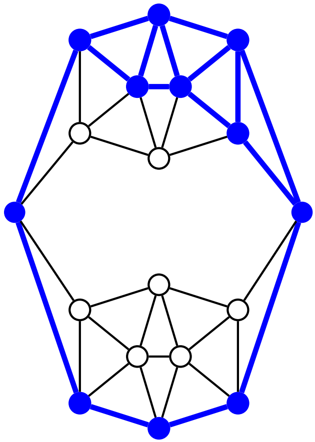

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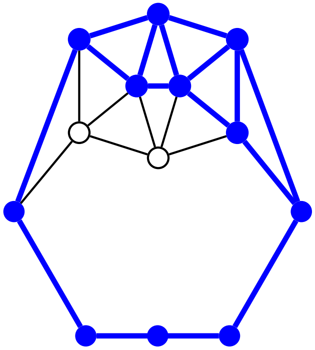

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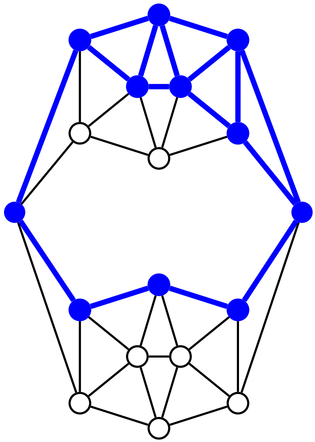

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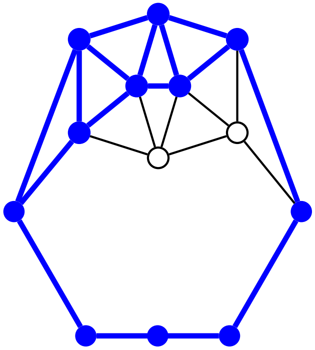

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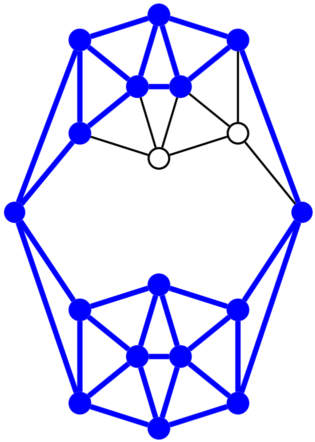

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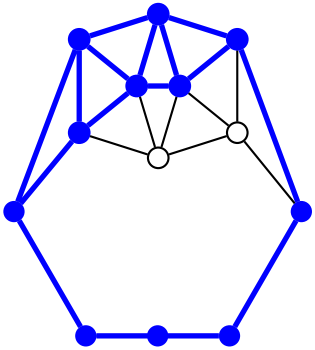

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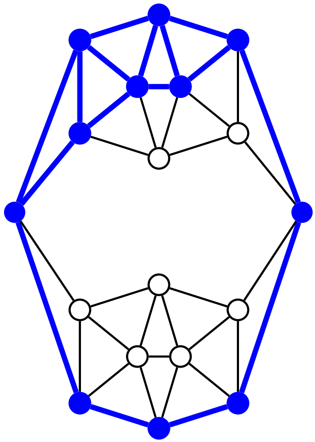

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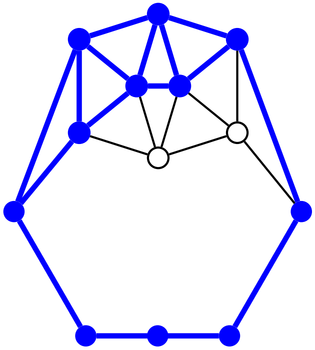

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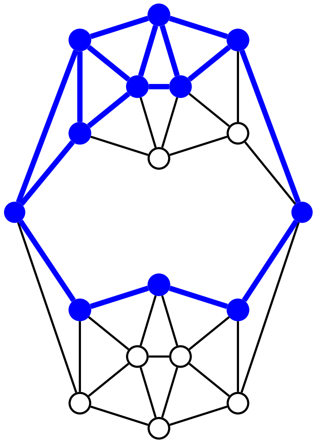

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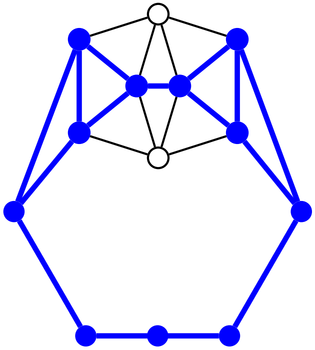

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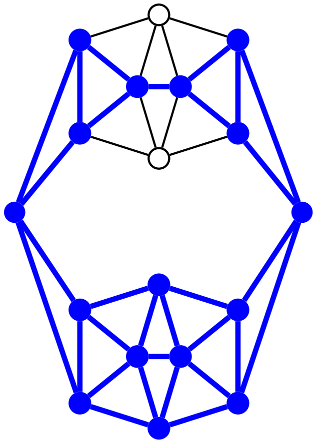

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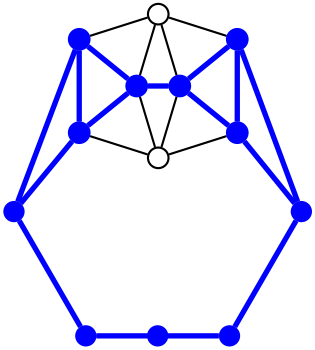

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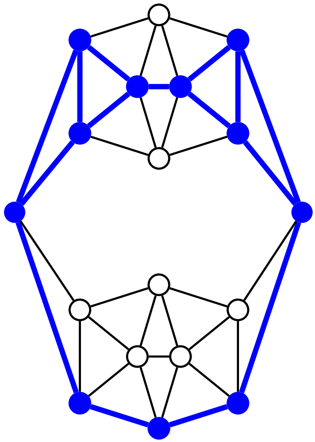

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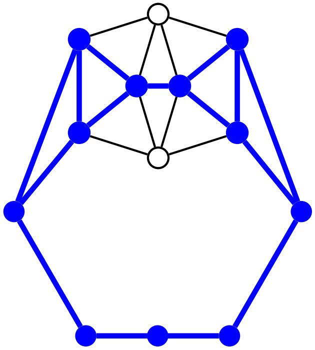

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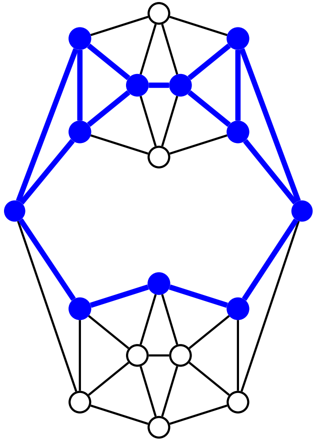

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$\mathcal{G}_H$ graphs

Given a graph H, define E_H^* as the set of its simplicial edges, and let a g-operation on $e \in E_H^*$ be either a gear composition or an edge subdivision applied on e. A graph G belongs to $\mathcal{G}_H$ if and only if

- *either* $G = H$,
- *or* $G = (L, B, e)$, where $L \in \mathcal{G}_H$, B is a gear, and $e \in E_H^* \cap E_L$, i.e., e is a simplicial edge of H on which no g-operations has been applied,
- *or* $G = L^e$, where $L \in \mathcal{G}_H$ and $e \in E_H^* \cap E_L$.

G-perfect graphs

A facet defining inequality $(\gamma, \gamma_0) \in \mathcal{G}$ if and only if it is (the sequential lifting of)

- either a rank inequality,
- or a 5-wheel inequality,
- or a geared or a g-lifted inequality associated with an inequality in $\mathcal{G}$.

A graph G is $\mathcal{G}$-perfect if and only if $STAB(G)$ can be described by inequalities in $\mathcal{G}\}$.

Theorem. Let H be a graph and E^*_H the set of its simplicial edges. If H and H^F are $\mathcal{G}$-perfect for any $F \subseteq E^*_H$, then every graph $G \in \mathcal{G}_H$ is $\mathcal{G}$-perfect.

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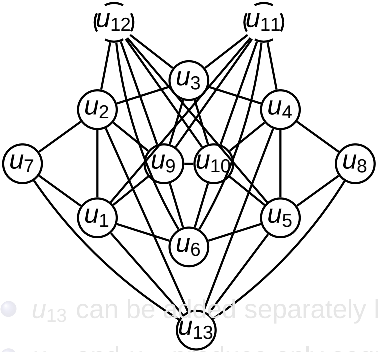
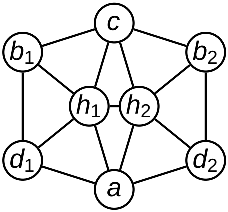
XX strips & gears





- u_{13} can be added separately by a proper linear strip
- u_{11} and u_{12} produce only sequential lifting of geared or g-lifted inequalities
  (+ two new g-lifted inequalities that are isomorphic to H^e)
- XX-strip composition and gear composition are equivalent
  (provided that the simplicial edge $\{v_1, v_2\}$ is such that $N(K_1 \cap K_2) \subseteq N(K_1) \cup N(K_2)$)

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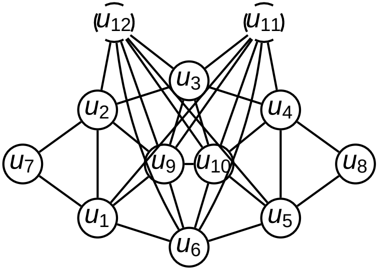

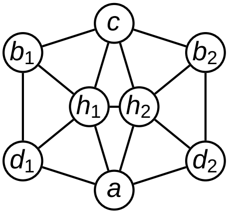

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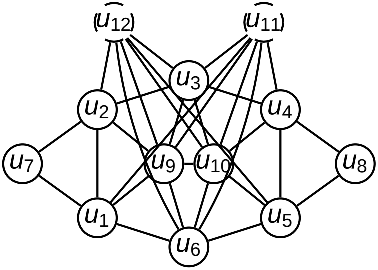

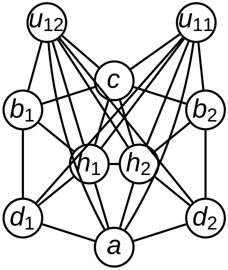

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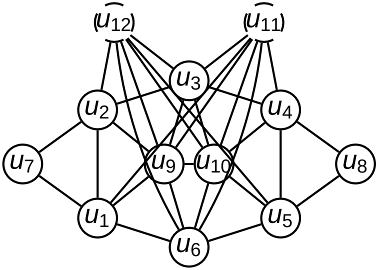

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Theorem. XX-graphs are $\mathcal{G}$-perfect.