Simultaneous Embedding with Fixed Edges

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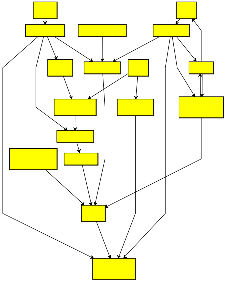
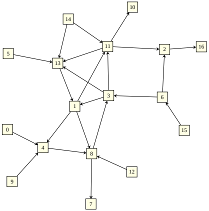
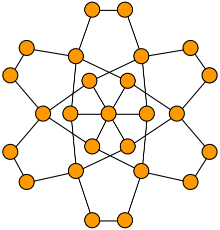
University of Cologne

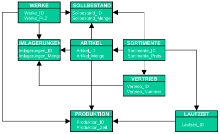
Aussois 2009

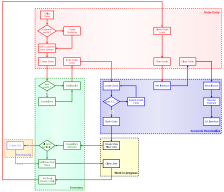
Graph Drawing











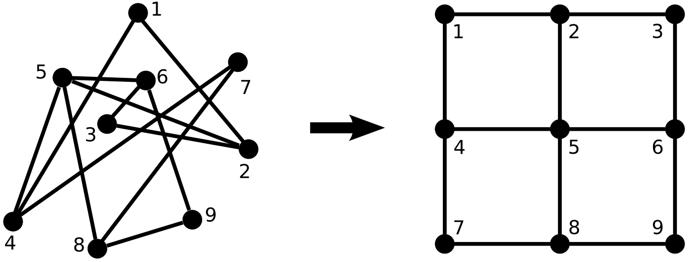
Input

One graph G

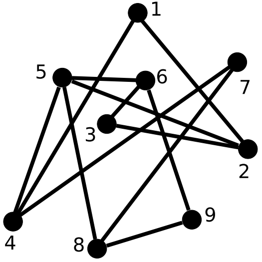
Output

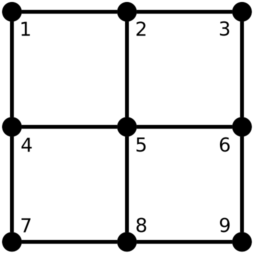
Layout of G

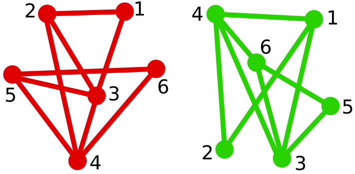
Graph Drawing



Simultaneous Graph Drawing





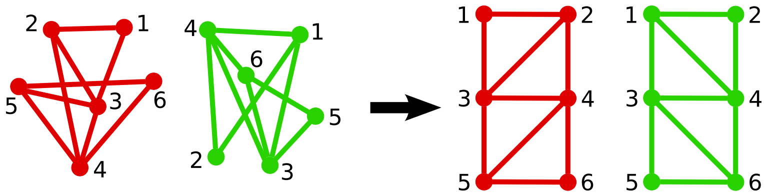


?

Definition

A *simultaneous embedding with fixed edges (SEFE)* of graphs $G_1, \ldots, G_k$ consists of drawings $\Gamma_1, \ldots, \Gamma_k$ with

- Γ_i is a planar drawing of G_i,
- every node in $G_i \cap G_j$ is drawn equally in Γ_i and Γ_j and
- every edge in $G_i \cap G_j$ is drawn equally in Γ_i and Γ_j.



Known results

Positive results

Garantueed SEFE for

- (tree, path) [Erten and Kobourov 2004]
- (outerplanar graph, cycle) [Di Giacomo and Liotta 2005]
- (planar graph, tree) [Frati 2006]

Negative result

Example pair without SEFE for

- two outerplanar graphs [Frati 2006]

NP-completeness

Theorem

To decide SEFE for three graphs is NP-complete.

Open problem

The complexity for two graphs.

P_{SEFE} - Definition

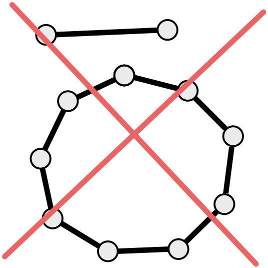
Definition

Let P_{SEFE} be the set of all planar graphs, that share a SEFE with any planar graph.

$$G_1 \in P_{SEFE},\ G_2 \text{ planar} \quad \Rightarrow \quad (G_1, G_2) \text{ has SEFE.}$$

Theorem

P_{SEFE} is the set of all planar graphs that do not contain the node-disjoint union of a cycle and an edge.



P_{SEFE} - Characterization 1 - Proof - Part 1

Theorem

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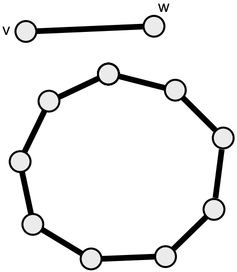

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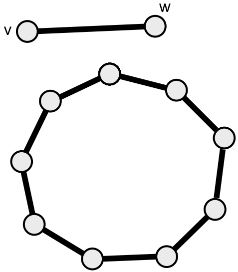

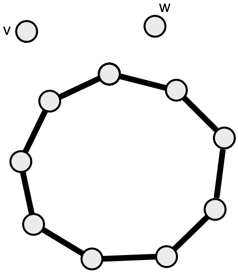

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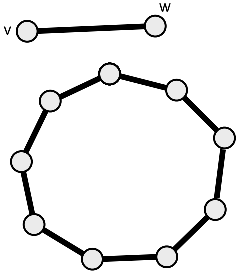

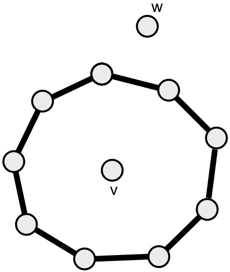

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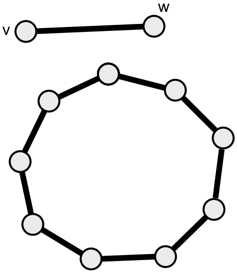
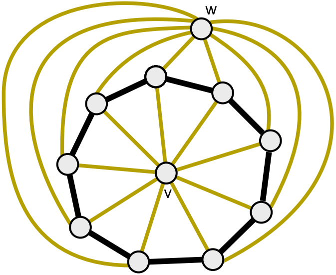
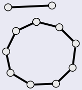




P_{SEFE} - Characterization 1 - Proof - Part 2

Construction SEFE:

G_1 planar, $G_2 \not\supseteq$ 

1. Planar drawing D_1 of G_1
2. Start D_2 of G_2 with $G_1 \cap G_2$
3. Insert remaining edges of G_2

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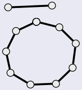
G_1 planar, $G_2 \not\supseteq$ 

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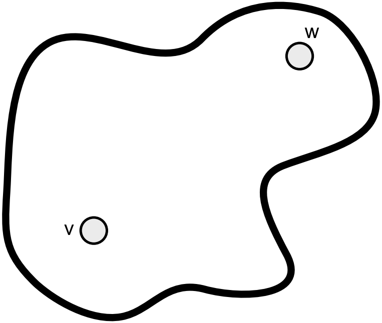

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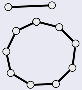
G_1 planar, $G_2 \not\supseteq$ 

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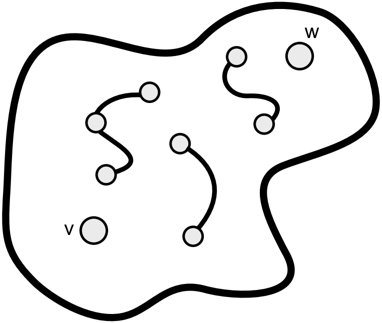

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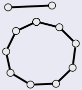
G_1 planar, $G_2 \not\supseteq$ 

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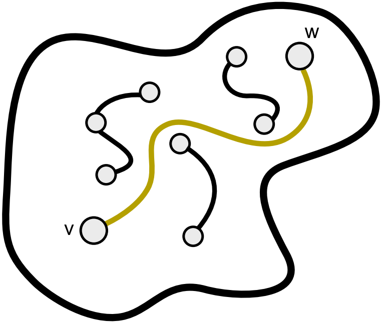
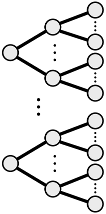


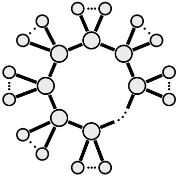
P_{SEFE} - Characterization 2

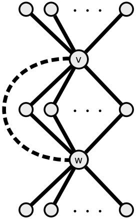
Theorem

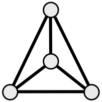
P_{SEFE} consists of all

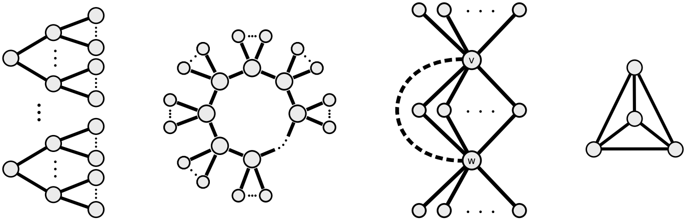
- *forests,*
- *circular caterpillars,*
- *bi-stars, and*
- *subgraphs of K_4.*



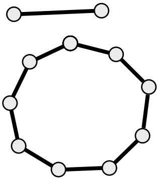








do not contain



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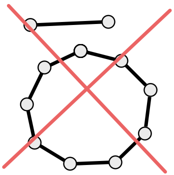
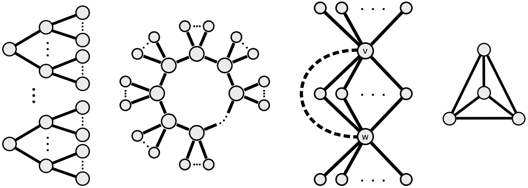


$\Longrightarrow$



Proof: Case distinction

Thanks.

Thank you very much for your attention.