On solving the multi-period location-assignment problem under uncertainty

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Outline

Universidad Rey Juan Carlos

The MISFLP problem

Given a time horizon, a set of customers and a set of facilities (e.g., production plants),
Multi-period Incremental Service Facility Location Problem (MISFLP)
is concerned with:

- locating the facilities within a given discrete set of potential sites and
- assigning the customers to the facilities along given periods in a time horizon.

The MISFLP problem

Assumptions

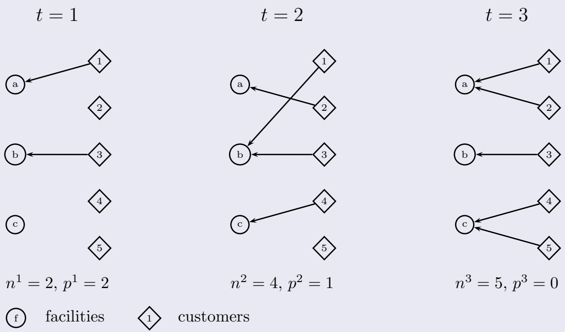
- Ensuring at each single period t the service of a minimum number of customers, say n^t.
- The allocation of any customer to the servers might change in different periods.
- Once a customer is served in a time period it must be served at any subsequent period.
- Once a facility is opened it remains open until the end of the time horizon.

Variation of the MISFLP problem

We present a variation of the MISFLP where:

- Each customer needs to be serviced only in a subset of the periods of the time horizon
- we assume that this set of periods is known for each customer
- There is uncertainty in some parameters of the problem

The problem



Consider a network including a set of *facilities* and a set of *customers*

The problem

- For the costumers:
  - The allocation of a customer to the facilities might change at different time periods but
  - Once a customer is assigned in a given time period, he must continue to be assigned to one facility.
  - A costumer cannot be assigned to more than one facility at each period
  - All customers must be found to have been assigned at the end of the time horizon

- At each single period
  - exactly p^t facilities are opened
  - at least n^t new customers are covered

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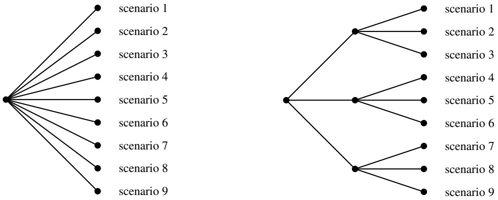
The problem

Costs

- Assigning a customer to a facility at a given period incurs a assignment cost, c_{ij}^t, even if the customer does not have a need for service in this period.
- There is a setup depreciation cost, f_i^t for the open facilities.
- the penalty cost, ρ_j, for the customers not served in time by the facilities

Uncertainty: Modeling via scenario tree



- Scenario is an execution of uncertain and deterministic parameters along different stages of the temporal horizon.
- Scenario group for a given stage is the set of scenarios with the same realization of the uncertain parameters up to the stage.
- Scenario tree scheme is a technique used to model and interpret the uncertainty.

Uncertainty in the problem

The most important uncertainty that we find in this problem is the number of costumers that will need to be serviced in each time period

Parameters

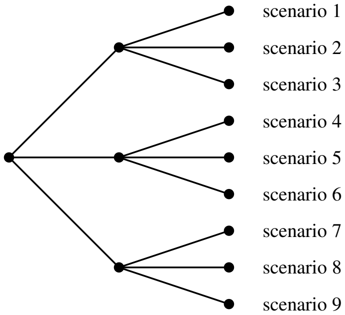
d_j^g, coefficient that takes the value 1 or 0 depending on whether or not customer j is available for being serviced at time period $t(g)$ under scenario group g, $\forall j \in \mathcal{J}$.

n^g, minimum number of customers to be serviced in time period $t(g)$ under scenario group g.

Variables

Non-anticipativity principle (*Rockafellar and Wets*)

If two different scenarios s and $s^{'}$ are identical until stage t as to as the disponible information in that stage, then the decisions (variables) in both scenarios must be the same too until stage t.



Impulse-Step variables based formulation

0–1 variables

$$y_i^g = \begin{cases} 1, & \text{if facility } i \text{ is open } \textbf{by} \text{ time period } t(g) \text{ under scenario group } g \\ 0, & \text{otherwise} \end{cases} \qquad \forall i \in \mathcal{I}, g \in \mathcal{G} : t(g) \in \mathcal{T}^*$$

and

$$x_{ij}^g = \begin{cases} 1, & \text{if customer } j \text{ is assigned to facility } i \textbf{ at} \text{ time period } t(g) \text{ under scenario group } g \\ 0, & \text{otherwise} \end{cases} \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}, g \in \mathcal{G}^-.$$

where $\mathcal{T}^* = \{t \in \mathcal{T} : t \leq |\mathcal{T}| - \tau\}$ and $\mathcal{G}^- \equiv \mathcal{G} \setminus \{0\}$

Note

The x–variables still are impulse variables, but the y–variables are step variables.

Pure 0–1 Model

Objective function

$$\min \sum_{i\in\mathcal{I}} f_i^0 y_i^0 + \sum_{g\in\mathcal{G}^-} w^g \left[\sum_{i\in\mathcal{I}}\left(f_i^{t(g)}(y_i^g - y_i^{\gamma(g)}) + \sum_{j\in\mathcal{J}} c_{ij}^{t(g)} x_{ij}^g\right) + \sum_{j\in\mathcal{J}} \rho_j d_j^g (1 - \sum_{i\in\mathcal{I}} x_{ij}^g)\right] =$$

$$\sum_{g\in\mathcal{G}}\sum_{j\in\mathcal{J}} w^g \rho_j d_j^g + \min \sum_{i\in\mathcal{I}}\left[\sum_{g\in\mathcal{G}:t(g)\in\mathcal{T}*} w^g (f_i^{t(g)} - f_i^{t(g)+1}) y_i^g + \sum_{g\in\mathcal{G}^-} w^g \sum_{j\in\mathcal{J}} \overline{c}_{ij}^g x_{ij}^g\right] \quad (1)$$

Note

where $\overline{c}_{ij}^g = c_{ij}^{t(g)} - \rho_j d_j^g$

$\gamma(g)$, inmediate scenario group to group g

Pure 0–1 Model

Objective function

$$\min \sum_{i\in\mathcal{I}} f_i^0 y_i^0 + \sum_{g\in\mathcal{G}^-} w^g \left[\sum_{i\in\mathcal{I}}\left(f_i^{t(g)}(y_i^g - y_i^{\gamma(g)}) + \sum_{j\in\mathcal{J}} c_{ij}^{t(g)} x_{ij}^g\right) + \sum_{j\in\mathcal{J}} \rho_j d_j^g \left(1 - \sum_{i\in\mathcal{I}} x_{ij}^g\right)\right] =$$

$$\sum_{g\in\mathcal{G}}\sum_{j\in\mathcal{J}} w^g \rho_j d_j^g + \min \sum_{i\in\mathcal{I}}\left[\sum_{g\in\mathcal{G}:t(g)\in\mathcal{T}*} w^g (f_i^{t(g)} - f_i^{t(g)+1}) y_i^g + \sum_{g\in\mathcal{G}^-} w^g \sum_{j\in\mathcal{J}} \overline{c}_{ij}^g x_{ij}^g\right] \quad (1)$$

Note

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Pure 0–1 Model

Constraints

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} x_{ij}^g \geq n^g \qquad \forall g \in \mathcal{G}^- : t(g) < |\mathcal{T}| \tag{2}$$

$$\sum_{i \in \mathcal{I}} x_{ij}^g \leq 1 \qquad \forall j \in \mathcal{J}, g \in \mathcal{G}^- : t(g) < |\mathcal{T}| \tag{3}$$

$$\sum_{i \in \mathcal{I}} x_{ij}^g = 1 \qquad \forall j \in \mathcal{J}, g \in \mathcal{G}_{|\mathcal{T}|} \tag{4}$$

$$\sum_{i \in \mathcal{I}} x_{ij}^{\gamma(g)} \leq \sum_{i \in \mathcal{I}} x_{ij}^g \qquad \forall j \in \mathcal{J}, g \in \mathcal{G} : t(g) > 1 \tag{5}$$

$$x_{ij}^g \leq y_i^{\gamma^k(g)} \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}, g \in \mathcal{G}^-, \text{ where } k = \min\{t(g), \tau\} \tag{6}$$

$$\sum_{i \in \mathcal{I}} (y_i^g - y_i^{\gamma(g)}) = p^t \qquad \forall g \in \mathcal{G}^- : t(g) \in \mathcal{T}^* \tag{7}$$

$$\sum_{i \in \mathcal{I}} y_i^0 = p^0 \tag{8}$$

$$y_i^{\gamma(g)} \leq y_i^g \qquad \forall i \in \mathcal{I}, g \in \mathcal{G}^- : t(g) \in \mathcal{T}^* \tag{9}$$

$$x_{ij}^g \in \{0,1\} \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}, g \in \mathcal{G}^- \tag{10}$$

$$y_i^{tg} \in \{0,1\} \qquad \forall i \in \mathcal{I}, g \in \mathcal{G} : t(g) \in \mathcal{T}^* \tag{11}$$

Splitting variables representation

For a general problem:

$$
\begin{array}{llllll}
\min & \sum\limits_{\omega\in\Omega} p^{\omega}\left(\mathbf{c}^T\mathbf{x}^{\omega} + \mathbf{q}^{\omega\,T}\mathbf{y}^{\omega}\right) & & & & \\
\text{s. t.} & \mathbf{A}\mathbf{x}^{\omega} & & = \mathbf{b} & & \\
 & \mathbf{T}^{\omega}\mathbf{x}^{\omega} & + \mathbf{W}\mathbf{y}^{\omega} & = \mathbf{h}^{\omega} & & \forall\omega\in\Omega \\
 & \mathbf{x}^{\omega} - \mathbf{x}^{\omega+1} & & = \mathbf{0} & & \forall\omega\in\Omega \\
 & \mathbf{x}^{\omega}, & \mathbf{y}^{\omega} & \in \{0,1\} & & \forall\omega\in\Omega
\end{array}
$$

$$
\begin{array}{cccccccccccccl}
\mathbf{x}^1 & \mathbf{y}^1 & \mathbf{x}^2 & \mathbf{y}^2 & \mathbf{x}^3 & \mathbf{y}^3 & \mathbf{x}^4 & \mathbf{y}^4 & \mathbf{x}^5 & \mathbf{y}^5 & \mathbf{x}^6 & \mathbf{y}^6 & & \\
\boxed{\mathbf{A}} & & & & & & & & & & & & & =\mathbf{b} \\
\boxed{\mathbf{T}^1} & \boxed{\mathbf{W}} & & & & & & & & & & & & =\mathbf{h}^1 \\
\mathbf{I} & & -\mathbf{I} & & & & & & & & & & & =\mathbf{0} \\
 & & \boxed{\mathbf{A}} & & & & & & & & & & & =\mathbf{b} \\
 & & \boxed{\mathbf{T}^2} & \boxed{\mathbf{W}} & & & & & & & & & & =\mathbf{h}^2 \\
 & & \mathbf{I} & & -\mathbf{I} & & & & & & & & & =\mathbf{0} \\
 & & & & \boxed{\mathbf{A}} & & & & & & & & & =\mathbf{b} \\
 & & & & \boxed{\mathbf{T}^3} & \boxed{\mathbf{W}} & & & & & & & & =\mathbf{h}^3 \\
 & & & & \mathbf{I} & & -\mathbf{I} & & & & & & & =\mathbf{0} \\
 & & & & & & \boxed{\mathbf{A}} & & & & & & & =\mathbf{b} \\
 & & & & & & \boxed{\mathbf{T}^4} & \boxed{\mathbf{W}} & & & & & & =\mathbf{h}^4 \\
 & & & & & & \mathbf{I} & & -\mathbf{I} & & & & & =\mathbf{0} \\
 & & & & & & & & \boxed{\mathbf{A}} & & & & & =\mathbf{b} \\
 & & & & & & & & \boxed{\mathbf{T}^5} & \boxed{\mathbf{W}} & & & & =\mathbf{h}^5 \\
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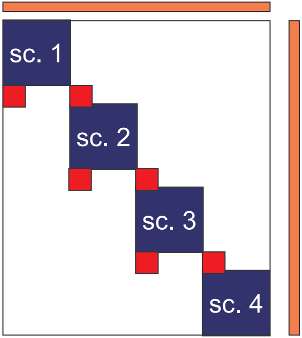
 & & & & & & & & & & \boxed{\mathbf{T}^6} & \boxed{\mathbf{W}} & & =\mathbf{h}^6 \\
-\mathbf{I} & & & & & & & & & & \mathbf{I} & & & =\mathbf{0}
\end{array}
$$

Branch-and-Fix Coordination

- *Non-anticipativity* constraints are relaxed:



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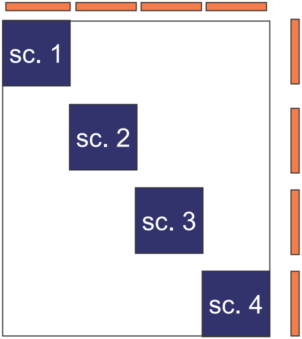

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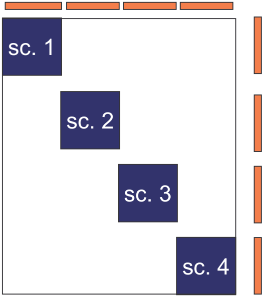
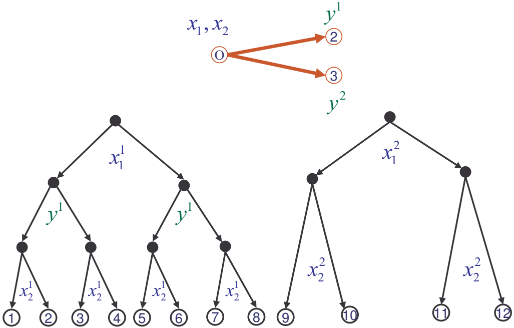


Independent problems

Each problem is solved by a Branch and Fix, coordinatting the decisions



Fix-and-Relax Coordination

Fix-and-Relax

- Introduced by Dillebenger et al. in 1994. See also Escudero and Salmerón in 2005.
- It is an heuristic approach to solve multi-level linear integer problems:
  - ✓ Initially only variables of the first level are 0-1 defined.
  - ✓ In successive stages, previous level variables are fixed, and current level variables are declared integer.

Fix-and-Relax Coordination (FRC)

- Combine Fix-and-Relax with Branch-and-Fix Coordination:
  - ✓ Non-anticipativity conditions are relaxed.
  - ✓ Each scenario group is solved by Fix and Relax.
  - ✓ Decisions are coordinated via Branch-and-Fix Coordination approach.

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FRC scheme

Fix-and-Relax submodel

- Given $\mathcal{V}_1 \dots \mathcal{V}_K$ a partition of K elements of the set of the variables $\mathcal{V}$, problem

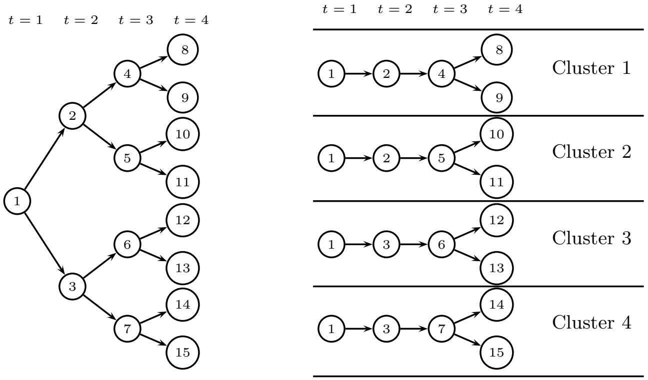
$$
\begin{aligned}
IP: \quad & \min_{\mathbf{x}\in\mathcal{X}} \mathbf{cx} \\
& \text{s. t. } \mathbf{x}_j \in \{0,1\} \qquad \forall j \in \mathcal{V}_k,\ k = 1,\dots,K,
\end{aligned}
$$

- This problem can be approximated by the model

$$
\begin{aligned}
IP^k: \quad & \min_{\mathbf{x}\in\mathcal{X}} \mathbf{cx} \\
\text{s. t.} \quad & \mathbf{x}_j = \overline{\mathbf{x}}_j \qquad \forall j \in \mathcal{V}'_k,\ k' < k, \\
& \mathbf{x}_j \in \{0,1\} \qquad \forall j \in \mathcal{V}_k, \\
& \mathbf{x}_j \in [0,1] \qquad \forall j \in \mathcal{V}'_k,\ k < k',
\end{aligned}
$$

It is the so-called FR level k

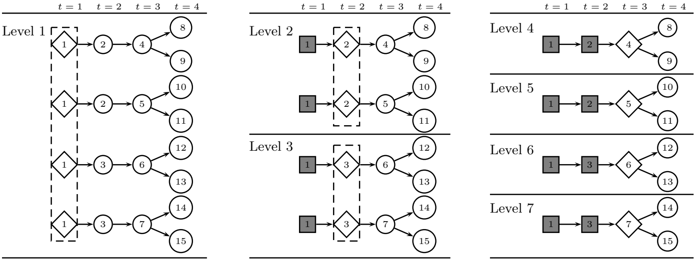
FR levels

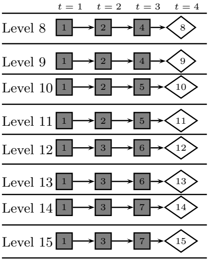


(a) Scenario tree

(b) Cluster structure

FRC scheme





Nodes whose non-anticipativity constraints are not relaxed

n Nodes whose variables have been 0-1 fixed

n Nodes whose variables have been 0-1 integer defined

n Nodes whose variables have been 0-1 continuous defined

Branching strategy

- We have chosen the *depth first* strategy for the *TNF* branching selection
- The criterion for branching consists of choosing the candidate *TNF* with the smallest Lagrangean *Substitution* value among the two sons of the last branched *TNF*.
- We have chosen two *largest small deterioration* strategies for the selection of the branching variable.

On candidate TNF bounding

- The bounding of a given candidate TNF, $\mathcal{J}_f, f \in \mathcal{F}$, can be obtained by using *Lagrangian Decomposition (LD)*:

$$\begin{aligned} Z_D(\mu) = \min \sum_{j \in \mathcal{J}_f} w^j(\mathbf{c}^j \mathbf{x}^j + \mathbf{a}^j \mathbf{y}^j) + \sum_{j \in \mathcal{J}_f} \mu^j(\mathbf{x}^j - \mathbf{x}^{j+1}) & \\ \text{s. t. } \mathbf{A}\mathbf{x}^j + \mathbf{B}\mathbf{y}^j = \mathbf{b}^j & \quad \forall j \in \mathcal{J}_f \\ \mathbf{0} \leq \mathbf{x}^j \leq 1, \ \mathbf{y}^j \geq \mathbf{0} & \quad \forall j \in \mathcal{J}_f, \end{aligned}$$

- Our aim is to obtain the bound $Z_D(\mu^*)$, where

$$\mu^* = \text{argmax}\{Z_D(\mu)\}.$$

Note

The number of Lagrange multipliers depends on the number of non yet branched on/fixed *common* variables in vector x^j and the number of nodes, $|\mathcal{J}_f|$, in the family.

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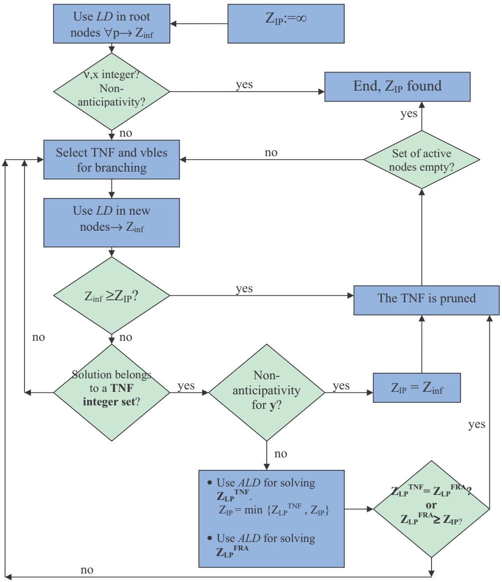
BFC algorithm



Why we use this modelization for the problem?

- A study of the deterministic problem has been done
- We have done a computational comparison among:
  1. Impulse variables based model
  2. Step variables based model
  3. Impulse-step variables based model

Computational Experience Deterministic Version

- Implemented in experimental C++ code.
- CPLEX v11 for solving each instance.
- Pentium IV, 1.8Ghz, 512 RAM
- Microsoft Visual Studio C++ compiler v6.0.

$\lvert\mathcal{J}\rvert$	$\lvert\mathcal{I}\rvert$	$\lvert\mathcal{T}\rvert$
50	30	4
50	30	8
50	30	12
100	30	4
100	30	8
100	30	12

10 instances for each row:

Total **60 instances**

Models' dimensions

	Impulse model				Impulse-step model				Step model			
Instance	*nv*	*nr*	*nel*	*dens* (%)	*nv*	*nr*	*nel*	*dens* (%)	*nv*	*nr*	*nel*	*dens* (%)
E4-P30-C50	6120	6388	42240	0.108	6120	6448	33390	0.085	6120	12048	30290	0.041
E8-P30-C50	12240	12796	111480	0.071	12240	12976	69870	0.044	12240	24176	60770	0.021
E12-P30-C50	18360	19204	204720	0.058	18360	19504	106350	0.030	18360	36304	91250	0.014
E4-P30-C100	12120	12738	84240	0.055	12120	12798	66390	0.043	12120	23998	60190	0.021
E8-P30-C100	24240	25546	222480	0.036	24240	25726	138870	0.022	24240	48126	120670	0.010
E12-P30-C100	36360	38354	408720	0.029	36360	38654	211350	0.015	36360	72254	181150	0.006

Computational experience

Table 1: Models' performance (1)

Instance	Z_{LP}	Impulse model $\overline{Z}_{IP}$	GAP	nn	tt	Impulse-step model $\overline{Z}_{IP}$	GAP	nn	tt	Step model $\overline{Z}_{IP}$	GAP	nn	tt
E4-P30-C50-1	3342.98	3345.64	0.00	0	2	3345.64	0.00	0	2	3345.64	0.00	3	9
E4-P30-C50-2	2247.29	2264.57	0.00	15	7	2264.57	0.00	9	8	2264.57	0.00	69	15
E4-P30-C50-3	2783.89	2836.86	0.00	172	36	2836.86	0.00	53	27	2836.86	0.00	136	69
E4-P30-C50-4	4038.62	4117.57	0.00	84	30	4117.57	0.00	89	32	4117.57	0.00	85	173
E4-P30-C50-5	3278.40	3555.76	0.00	26783	2930	3555.76	0.00	8486	1212	3567.90	3.39	35685	7200
E4-P30-C50-6	3292.64	3307.04	0.00	8	10	3307.04	0.00	11	8	3307.04	0.00	52	15
E4-P30-C50-7	3975.75	3979.51	0.00	0	5	3979.51	0.00	0	5	3979.51	0.00	0	7
E4-P30-C50-8	3575.80	3709.69	0.00	3754	483	3709.69	0.00	1348	253	3709.69	0.00	9572	1581
E4-P30-C50-9	5017.59	5318.30	0.00	1246	573	5318.30	0.00	1431	642	5318.30	0.00	9248	3719
E4-P30-C50-10	3053.63	3054.02	0.00	0	1	3054.02	0.00	0	0	3054.02	0.00	0	2
E8-P30-C50-1	4987.80	5493.71	4.20	13797	7200	5459.18	0.00	5190	5155	5582.31	7.14	6422	7200
E8-P30-C50-2	6623.97	7156.99	4.07	11453	7200	7082.90	1.69	9213	7200	7152.06	4.26	5860	7200
E8-P30-C50-3	4535.72	4543.95	0.00	13	16	4543.95	0.00	12	13	4543.95	0.00	24	29
E8-P30-C50-4	5076.35	5110.59	0.00	364	89	5110.59	0.00	109	68	5110.59	0.00	463	187
E8-P30-C50-5	5170.15	5663.79	6.17	15390	7200	5593.65	1.95	13268	7200	5725.99	7.59	7884	7200
E8-P30-C50-6	5281.55	5393.55	0.00	2377	1049	5393.55	0.00	788	564	5393.55	0.00	2573	1521
E8-P30-C50-7	7263.21	8331.34	9.77	8548	7200	8062.83	5.39	7060	7200	8309.64	11.21	5067	7200
E8-P30-C50-8	7263.10	7818.56	3.64	11534	7200	7732.08	0.00	5149	3504	7832.18	4.92	5078	7200
E8-P30-C50-9	9689.74	10674.56	8.40	7017	7200	10770.25	7.23	6127	7200	10816.35	9.30	3197	7200
E8-P30-C50-10	6011.37	6250.15	1.06	20978	7200	6238.55	0.00	30929	5543	6238.55	0.00	6041	4051
E12-P30-C50-1	8889.07	9847.44	9.10	5091	7200	9684.71	4.29	3633	7200	10502.29	16.74	2487	7200
E12-P30-C50-2	8439.09	8887.76	3.40	10400	7200	8799.24	0.20	12148	7200	8807.30	1.96	2953	7200
E12-P30-C50-3	10424.14	12082.32	15.48	1110	7200	12255.81	15.96	2039	7200	13612.29	28.89	1419	7200
E12-P30-C50-4	12843.60	14514.53	11.72	935	7200	14736.23	14.38	1081	7200	15755.90	22.24	937	7200
E12-P30-C50-5	8589.67	9311.07	6.50	6541	7200	9275.67	5.30	5420	7200	9597.19	10.12	2784	7200
E12-P30-C50-6	10929.99	12203.15	10.22	2489	7200	12024.37	7.00	3918	7200	12594.58	13.92	2131	7200
E12-P30-C50-7	7689.29	8306.83	6.83	6888	7200	8296.32	5.70	6348	7200	8558.81	9.84	3421	7200
E12-P30-C50-8	8761.03	9737.51	10.08	3784	7200	9618.83	7.04	3949	7200	11994.70	35.54	2082	7200
E12-P30-C50-9	7464.05	8110.70	6.50	6486	7200	8141.92	5.91	7125	7200	8181.13	8.50	2808	7200
E12-P30-C50-10	8872.70	9233.33	1.93	8512	7200	9233.33	1.03	7063	7200	9270.11	1.83	2843	7200

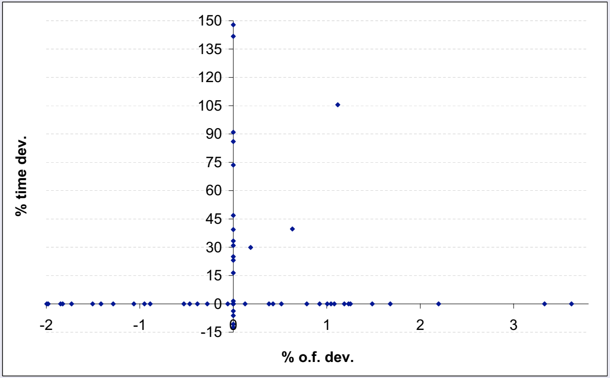
Computational experience

Table 2: Models' performance (2)

Instance	Z_{LP}	Impulse model $\overline{Z}_{IP}$	GAP	nn	tt	Impulse-step model $\overline{Z}_{IP}$	GAP	nn	tt	Step model $\overline{Z}_{IP}$	GAP	nn	tt
E4-P30-C100-1	12531.57	13847.01	7.16	7842	7200	13885.56	5.77	8468	7200	14061.53	9.34	2851	7200
E4-P30-C100-2	10398.54	10706.64	0.00	9062	3898	10706.64	0.00	6121	2246	10732.25	1.02	13656	7200
E4-P30-C100-3	11404.14	11459.41	0.00	49	76	11459.41	0.00	94	79	11459.41	0.00	143	83
E4-P30-C100-4	13444.33	13664.57	0.00	873	373	13664.57	0.00	126	72	13664.57	0.00	1119	1014
E4-P30-C100-5	6513.29	6907.84	1.07	17375	7200	6907.84	0.00	10147	2905	6910.58	3.56	9724	7200
E4-P30-C100-6	10685.03	10786.09	0.00	244	138	10786.09	0.00	245	136	10786.09	0.00	288	223
E4-P30-C100-7	12073.00	13096.48	3.83	9251	7200	13104.20	6.13	10004	7200	13175.03	5.65	4550	7200
E4-P30-C100-8	9704.35	9898.25	0.00	1010	646	9898.25	0.00	1720	555	9898.25	0.00	1152	951
E4-P30-C100-9	9819.83	10146.55	0.43	21086	7200	10146.55	0.00	17963	4903	10146.55	0.00	5090	3492
E4-P30-C100-10	5787.66	6311.46	5.20	15983	7200	6243.97	2.77	16124	7200	6305.82	6.77	9359	7200
E8-P30-C100-1	20970.42	23365.03	11.08	515	7200	23799.45	12.89	570	7200	31726.60	50.59	496	7200
E8-P30-C100-2	15041.27	16765.60	10.77	1031	7200	17107.00	11.55	2349	7200	17743.49	16.87	928	7200
E8-P30-C100-3	13141.88	14910.47	13.11	973	7200	14989.64	12.11	1536	7200	22476.08	68.96	908	7200
E8-P30-C100-4	11535.78	11675.96	0.31	8604	7200	11675.96	0.00	11283	5166	11678.06	0.70	3909	7200
E8-P30-C100-5	18725.57	19495.83	3.52	4928	7200	19317.47	2.39	4408	7200	19460.17	3.20	1619	7200
E8-P30-C100-6	14622.01	15176.99	2.12	6057	7200	15112.77	0.37	6613	7200	15278.84	3.33	2753	7200
E8-P30-C100-7	17036.63	17680.77	3.01	5230	7200	17590.28	1.89	5649	7200	17816.87	4.06	1737	7200
E8-P30-C100-8	10155.12	11405.18	11.71	2482	7200	11006.87	6.65	3286	7200	11475.06	12.38	1154	7200
E8-P30-C100-9	10743.58	11152.82	3.04	8217	7200	11065.77	1.45	7514	7200	11111.91	2.64	3003	7200
E8-P30-C100-10	17369.59	19286.00	10.87	1017	7200	19286.15	9.77	2375	7200	19442.26	11.34	1133	7200
E12-P30-C100-1	18698.49	21049.71	11.81	508	7200	20802.63	10.57	569	7200	21494.23	14.23	440	7200
E12-P30-C100-2	20774.55	23609.97	13.47	476	7200	23863.89	14.45	562	7200	31582.73	51.65	469	7200
E12-P30-C100-3	22567.11	25138.15	11.13	489	7200	25255.88	10.73	615	7200	27125.31	19.54	360	7200
E12-P30-C100-4	22480.45	25894.32	15.09	320	7200	26417.51	17.39	466	7200	39561.68	75.32	179	7200
E12-P30-C100-5	23316.14	28767.70	22.89	489	7200	25902.03	10.47	460	7200	28711.56	22.42	334	7200
E12-P30-C100-6	21771.70	24158.94	10.72	518	7200	23639.52	8.10	538	7200	35032.97	60.28	452	7200
E12-P30-C100-7	26148.93	28767.73	9.95	488	7200	29142.17	10.79	508	7200	36030.82	37.16	328	7200
E12-P30-C100-8	23013.60	26683.24	15.64	253	7200	27153.20	17.68	307	7200	35956.34	56.10	172	7200
E12-P30-C100-9	22047.82	24240.60	9.44	649	7200	24697.08	11.54	729	7200	27107.40	22.51	448	7200
E12-P30-C100-10	22588.37	24748.01	9.21	519	7200	24985.46	10.31	697	7200	29572.11	30.19	380	7200

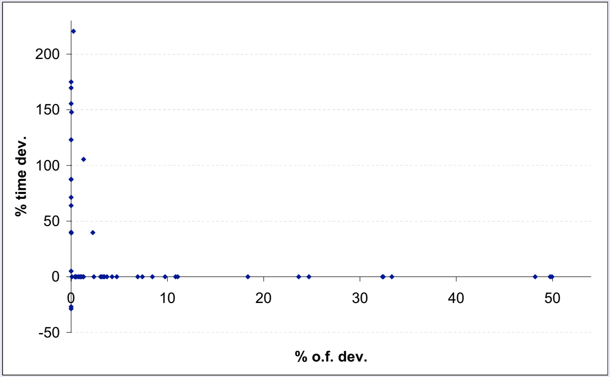
Graphical comparison of the results obtained with the different models

Model M1 with respect to Model M2



Graphical comparison of the results obtained with the different models

Model M3 with respect to Model M2



Conclusions and Future work

- A variation of the multi-period incremental service facility location problem has been presented.
- A variation with uncertainty in the demand and the set of periods when each customer requieres service is presented by using Stochastic Programming.
- Three 0–1 equivalent formulations are proposed for the deterministic models, based on the impulse and step variables approaches.
- An intensive computational experimentation has been performed for the deterministic models.
    - Impulse-Step variables based formulation shows better results.
- Work-in-progress:
    - Computational experience for the stochastic model